# When Kolm Meets Mirrlees: ELIE

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# When Kolm Meets Mirrlees: ELIE

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## 1 Introduction

Since the work by Mirrlees (1971, 1974, 1986), the second-best approach to welfarist optimal taxation has been widely adopted. This approach discards *personalized* lump-sum transfers and taxes because they are not implementable when the exogenous parameters on which they depend are private information. In contrast, in his recent book *Macrojustice* (Kolm, 2004), Serge-Christophe Kolm proposes a tax scheme derived from fundamental principles of justice that corresponds in essence to providing everyone with a common lump-sum subsidy and then taxing productivity linearly. The lump-sum subsidy is common to everyone, and so does not depend on private information. Nevertheless, because productivity is taxed in addition to the lump-sum transfer, the tax scheme proposed by Kolm depends on knowing an individual's productivity. Kolm claims that this schedule achieves justice without resulting in losses in efficiency. The practicality of this proposal is likely to be received with scepticism by the common public economist for whom it is like claiming to have solved the problem of squaring the circle.

However, the tax scheme derived by Kolm (2004) has many attractive features that make it worth further study. First, it turns out to have a remarkable structure when every individual to whom it applies provides a quantity of labour in excess of what is needed for him to pay his net tax. In this case, everyone transfers the income from the same time spent working to society in return for which they each receive the average amount transferred. In other words, each citizen works a fixed fraction of his time with a view to paying his contribution to the rest of society and is then free to devote

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his remaining time either to labour or leisure. This structure is referred to as *Equal-Labour Income Equalization* or ELIE by Kolm (2004, p. 112). ELIE is a very simple way of equally distributing among citizens contributions from them based on their means: "from each, to each other, the product of the same labour" and "from each, to each other, according to her capacities." From an economic policy perspective, the idea that time worked above some threshold, like the legal working hours per week, must be free of taxes has recently been defended by some economists (e.g., Godet, 2007) and has been partially implemented in France by President Sarkozy.

Formally, an *ELIE tax scheme* consists of (i) a tax based on labour productivity and also (ii) an *endogeneity* condition on individual labour supply. Therefore, to be as clear and precise as possible, the tax in (i) should be distinguished from the ELIE tax scheme itself. The former consists of a lump-sum tax paid to the government that is proportional to productivity combined with a transfer from the government of the average amount paid by all individuals. The net taxes paid by all of the individuals is necessarily budget balanced when the social objective is purely redistributive. The formula specifying the net tax in (i) is referred to as *Kolm's formula*. Given this definition, any ELIE tax scheme must belong to the family of Kolm formula tax schemes. That is why studying Kolm formula tax schemes in conjunction with ELIE tax schemes seems interesting to us. Nevertheless, it should be clearly emphasized that Kolm (2004) only argues in favour of ELIE, which consists of both (i) and (ii).

Given these caveats, this article aims at casting light on Kolm's formula and ELIE from the viewpoint of Mirrleesian welfarist optimal taxation so as to address some issues that are not fully embraced in Kolm's work. It endeavours to understand how ELIE tax schemes can be interpreted in this standard framework. We focus on three issues.

(1) The endogeneity condition (ii) that must be added to a Kolm formula tax scheme to obtain ELIE basically depends on the labour responses of the utility maximizing individuals. Therefore, it needs to be checked that, when ELIE is put into practice, individuals have the incentive to work no less than the time required to pay the common contribution. As a consequence, some conditions have to be met to obtain ELIE, and it is thus worthwhile to determine if these conditions are restrictive. More specifically, it is shown that the range of possible redistributions is significantly reduced when the endogeneity condition is taken into account.

(2) Kolm's formula corresponds to a first-best solution to the problem of wealth redistribution within the population. Any Kolm formula tax scheme gives rise to a Pareto optimal allocation that can be obtained as the outcome of the maximization of a social welfare function for appropriate social weights. Uncovering the weights that generate Kolm's formula and ELIE is thus of crucial importance for the understanding of these redistributive mechanisms. The shape of the distribution of these weights proves to be very specific and in sharp contrast with what is normally assumed about the weight distribution

in the standard approach to optimal taxation. In particular, it is shown that these social weights must be strictly increasing with ability for the set of working individuals provided that some very weak conditions are satisfied. This result remains valid when there are two parameters of heterogeneity in the population: productivities on the one hand and the taste for consumption (or leisure) on the other.

(3) It is argued by Kolm (2010, p. 95) that ELIE is incentive-compatible in the sense that "it induces individuals to work with their capacities that are the most highly remunerated." This raises the question of whether these abilities are observable or can be inferred from publicly verifiable information, that is, whether the problem is first- or second-best. In his seminal article, Mirrlees (1971) popularized the idea that gross income (or total product) is observable, but that neither the time spent working nor ability (which is equal to the wage in his model) is observable. He says that "the government can observe the total product of each individual, that is the product of the wage rate and the amount worked, but is unable to observe either of these alone" (Mirrlees, 1997, p. 1316). It is the inability of the policy-maker to determine an individual's ability that accounts for the second-best nature of his problem. However, as emphasized by Mirrlees (1971, p. 208) himself in the conclusion of his article: "It would be good to devise taxes complementary to the income-tax, designed to avoid the difficulties that tax is faced with." Such a tax could be based on the ratio between the gross income and the hours worked by an individual if both of these variables are verifiable and can thus be included in the contract between the taxpayer and the policy-maker. The basic difference between the latter tax and a Mirrleesian income tax is thus the variables that are verifiable. Nevertheless, if an individual's gross income and hours worked are both observable, as pointed out by Dasgupta and Hammond (1980), it does not follow that one can infer her ability because this individual may choose to not work to the best of her ability.

We mainly consider two informational frameworks, those of Dasgupta and Hammond (1980) and Mirrlees (1971). In both cases, gross incomes are observable. In the former, hours worked are also observable, whereas in the latter, they are not. One might expect that the incentive-compatibility of ELIE is likely to depend on whether gross incomes and hours worked are verifiable by the tax authorities. When they are, ELIE is incentive compatible. Specifically, we show how the first-best net transfers of ELIE can be implemented by means of a truthful direct mechanism in weakly dominant strategies. In effect, these informational assumptions justify leaving the second best for the first best, with the consequence that ELIE resolves the fundamental trade-off between equity and efficiency. However, for individuals engaged in what we call "brainwork", tracking the time worked and work attendance is irrelevant. In this case, it is established that individuals have an incentive to misreport their productivities through the gross-income/labour combinations they choose.

Related work by Fleurbaey and Maniquet (2011) has also considered the incentive-compatibility of ELIE in both of the above-mentioned informational settings. A major difference between their analysis and ours is that they assume that individuals exhibit a diversity of preferences for leisure. Kolm regards this diversity as a private matter and not as a legitimate ground for redistribution, in contrast to productivity differences which are. Therefore, from an ethical viewpoint, it seems justified—as a first pass—to focus on the latter and to consider, as we do here, the restrictive framework in which all individuals have the same preferences. The other advantage of this approach is obviously that it avoids introducing a second source of heterogeneity into the Mirrlees model, which is well known to make it difficult to solve the optimal tax problem. To cope with this additional source of heterogeneity, Fleurbaey and Maniquet employ a different social welfare function from the utilitarian one used here, one that is less demanding in terms of the interpersonal utility comparisons that it requires. Using their framework, Fleurbaey and Maniquet provide an axiomatic foundation for ELIE to serve as a basic flat tax.

The rest of this article is organized as follows. Section 2 clarifies the difference between a Kolm formula tax scheme and ELIE. Section 3 investigates the restrictions under which the former coincides with the latter. Section 4 tackles the problem of deriving ELIE as a first-best tax scheme in the standard framework of optimal taxation. Section 5 focuses on the implementability of ELIE. Section 6 offers concluding comments.

## 2 ELIE: Type-Dependent Budget Sets and Corvée Labour

The population consists of a continuum of individuals who only differ in productivity $\theta$. Hence, an individual whose productivity is $\theta$ is referred to as a"$\theta$-individual". The technology exhibits constant returns to scale. There are two commodities, consumption $x \in \mathbb{R}_+$ and labour $\ell \in [0, 1]$, where the time endowment of each individual has been normalized to equal 1. The consumption good is chosen as the numeraire. A $\theta$-individual working $\ell$ units of time has gross income $z := \theta\ell$.

Individual productivity $\theta$ belongs to $\Theta \equiv \left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}_+$. Its cumulative distribution function $F\colon \Theta \to [0, 1]$, assumed to be continuously differentiable with derivative $f(\theta) > 0$, is common knowledge.

The tax policy can now be introduced formally. As stressed in the introduction, we are interested in two kinds of tax schedules.

- With a *Kolm formula tax scheme of degree $k$*, every $\theta$-individual is required to transfer $k\theta$ to society in exchange for which he receives $k\mathbf{E}[\theta]$, where $\mathbf{E}[\cdot]$ denotes the expectation over $\Theta$. Hence, the tax function is $T\colon \Theta \times [0, 1] \to \mathbb{R}$, where

$$T(\theta, k) = k(\theta - \mathbf{E}[\theta]). \quad (1)$$

- The *ELIE tax scheme of degree k* combines (i) a Kolm formula tax scheme of degree $k$ with (ii) a condition on the endogenous individual labour supply. In the absence of involuntary unemployment, this condition states that all productive individuals must provide $\ell \geq k$ to take part in the overall redistributive mechanism (Kolm, 2010, p. 102). Because it can be argued that a redistributive tax schedule should be universal with the same schedule applied to everyone, special attention will also be paid below to the case in which all citizens face the ELIE tax scheme of degree $k$.[1]

Given the tax function $T$, the *budget constraint* of a $\theta$-individual is

$$0 \leq x \leq \theta\ell - T(\theta, k) = \theta(\ell - k) + k\mathbf{E}[\theta]. \quad (2)$$

Because a $\theta$-individual cannot spend more on consumption than the maximum net income he obtains when devoting his whole time endowment to labour, his maximum possible consumption is equal to

$$x_{\max}(\theta, k) := \theta(1 - k) + k\mathbf{E}[\theta] \quad (3)$$

in the absence of exogenous wealth. For $k \neq 1$, this upper bound on individual consumption, $x_{\max}(\theta, k)$, is strictly increasing in $\theta$: the more productive an individual, the wider the range of consumption levels available to him. In addition, because net income cannot be negative, a $\theta$-individual chooses his labour supply in $[\ell_{\min}(\theta, k), 1]$, where

$$\ell_{\min}(\theta, k) := \max\left\{0, k\left(1 - \frac{\mathbf{E}[\theta]}{\theta}\right)\right\}. \quad (4)$$

Note that $\ell_{\min}(\theta, k)$ is always less than $k$. Given (2), (3), and (4), the budget set of a $\theta$-individual is defined as

$$\mathcal{B}(\theta) := \{(x, \ell) \in \mathbb{R}_+ \times [\ell_{\min}(\theta, k), 1] : x \leq \theta\ell + k(\mathbf{E}[\theta] - \theta)\}. \quad (5)$$

By definition, the budget set of a $\theta$-individual is independent of any endogeneity condition. It is thus the same with a Kolm formula tax scheme of degree $k$ and the corresponding ELIE tax scheme of degree $k$. Consequently, except in the laissez-faire case in which $k = 0$ and, thus, $\ell_{\min}(\theta, k) = 0$ for all $\theta$, the geometry of a Kolm formula tax scheme and ELIE basically depends on whether an individual's productivity is less than the average. This observation is illustrated in Figure 1 for a population consisting of three individuals whose productivities are chosen so that $\theta_1 < \mathbf{E}[\theta]$, $\theta_2 = \mathbf{E}[\theta]$, and

[1] These desiderata are incorporated in the actual tax schedules in many developed countries. In France, for instance, the 13th article of the Declaration of the Rights of Man and of the Citizen, which has a constitutional status, states that the common contribution should be equitably distributed among *all* the citizens in proportion to their means.

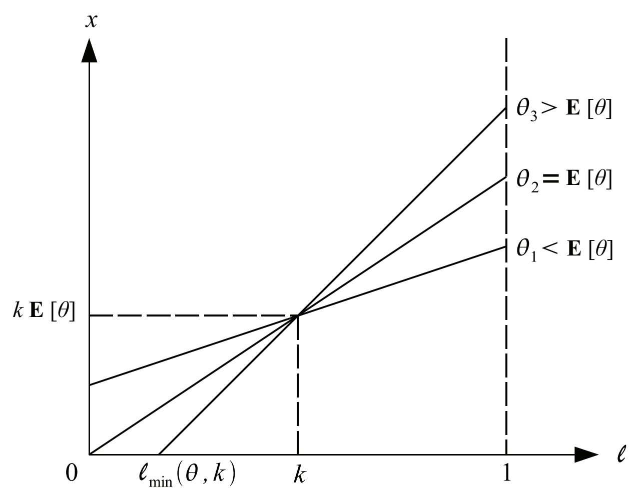


**Fig. 1** Budget sets

$\theta_3 > \mathbf{E}[\theta]$. The fact that an individual budget set is type-dependent with a *positive lower bound on labour supply* for the more productive part of the population (i.e., for $\theta > \mathbf{E}[\theta]$) is a fundamental feature of a Kolm formula tax scheme and ELIE.

The type-dependency of the individual budget sets distinguishes Kolm formula and ELIE tax schemes from the usual tax schedules considered in the second-best optimal income tax literature. Indeed, in this literature, all individuals make their choices from the same budget set in the gross income/consumption space. In contrast, Kolm's formula and ELIE gives rise to *inclusion of budget sets*, as illustrated in Figure 2.[2] In this diagram, the opportunity set of an individual of type $\theta_i$, $i = 1, 2, 3$, with $\theta_1 < \theta_2 < \theta_3$ consists of all the points in the nonnegative quadrant lying on or below the line marked $\theta_i$.

With regard to labour supply, a Kolm formula tax scheme and ELIE constrain every individual whose productivity is above the average to work at least $\ell_{\min}(\theta, k) > 0$ in order to pay the strictly positive tax $T(\theta, k) = k\,(\theta - \mathbf{E}[\theta])$. So, contrary to the situation of individuals in the less productive part of the population, highly skilled individuals do not have the *freedom of being idle*. This assertion must be qualified. Its validity rests on the assumption that highly skilled individuals have no assets and cannot borrow. If this assumption were relaxed, the model would no longer be static and intertemporal aspects should be taken into account.

[2] Kolm has shown that ELIE nevertheless exhibits equality of opportunity for some well-defined indexes of equality of opportunity.

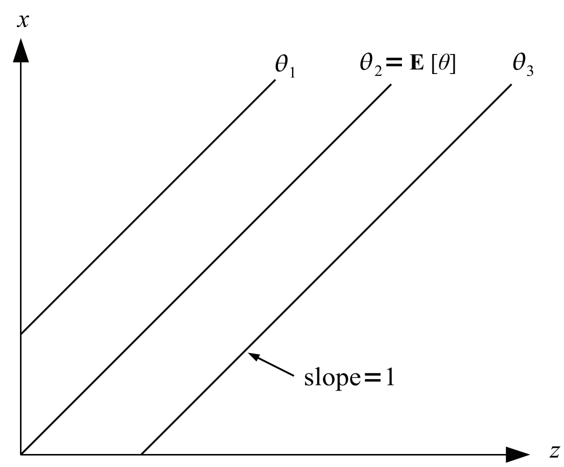


**Fig. 2** Inclusion of budget sets in the gross-income/consumption space

In the absence of intertemporal considerations, $\ell_{\min}(\theta, k)$ is increased from 0 to $1 - \mathbf{E}[\theta]/\overline{\theta}$ when $k$ goes from 0 to 1. Thus, $\ell_{\min}(\theta, k)$ can be regarded as the unpaid amount of labour required by Kolm's formula in lieu of taxes, i.e., as "*corvée labour*", with $k$ corresponding to the degree of "serfdom" of the talented.

An example is provided in Figure 3 for the case in which individuals have preferences represented by the log-transform of the Cobb-Douglas utility function

$$U(x, \ell) = \alpha \log x + (1 - \alpha) \log(1 - \ell). \tag{6}$$

We assume that $\alpha = 0.5$, productivity levels are uniformly distributed between 1 and 2, and $k = 0.25$. In this example, everyone provides an amount of labour $\ell(\theta, k)) > k$, which consists of corvée labour in the amount of $\ell_{\min}(\theta, k)$ and free labour in the amount of $\ell(\theta, k) - \ell_{\min}(\theta, k)$.

## 3 The Requirements of ELIE

Because the definition of ELIE includes a condition on the endogenous labour supply, it is difficult to see what the necessary conditions are for it to hold without using a microeconomic model. That is why it is worth examining the utility maximization programme of the individuals facing a Kolm formula tax scheme. It is maintained, in this section, that all individuals have the same preferences over consumption and leisure. They are represented by a

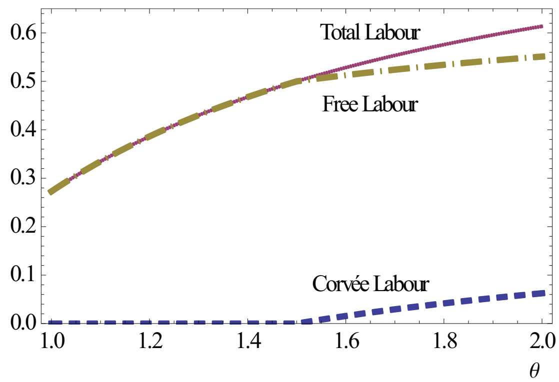


**Fig. 3** Corvée labour, free labour, total labour

twice continuously differentiable and strictly concave utility function $U : \mathbb{R}_+ \times [0,1] \to \mathbb{R}$, with $U'_x > 0$, $U'_\ell < 0$, and $U(x,\ell) \to -\infty$ when $x \to 0$ or $\ell \to 1$. The *utility maximization programme* of a given $\theta$-individual is thus

$$\max_{(x,\ell)\in\mathcal{B}(\theta)} U(x,\ell). \tag{7}$$

Let $\ell(\theta,k)$ denote the individual labour supply.

There are at least three ways of dealing with the endogeneity condition involved in the definition of the ELIE tax scheme of degree $k$: (i) the class of individual preferences could be restricted, (ii) each individual who would voluntarily choose to work less than the required common contribution could be excluded from the redistributive scheme, or (iii) the values of equalization labour $k$ could be restricted to those that are feasible. The third possibility seems the most natural to us. Formally, it amounts to determining *which values of k are compatible with the requirement that every individual provides at least k hours of labour*.

In order to cast light on the necessary restrictions on $k$ for this to be the case, it is assumed that all individuals have the same Cobb-Douglas preferences represented by (6) with $0 < \alpha < 1$. This utility specification is purely illustrative; other utility functions could naturally be considered. A $\theta$-individual chooses the consumption/labour bundle that solves the maximization problem in (6). The necessary and sufficient first-order condition implies that

$$\ell(\theta) = \max\left\{0, \alpha + (1-\alpha)\frac{T(\theta,k)}{\theta}\right\}, \tag{8}$$

$$x(\theta) = \begin{cases} \alpha(\theta - T(\theta, k)) & \text{if } \theta > \theta_0, \\ -T(\theta, k) & \text{if } \theta \leq \theta_0, \end{cases} \tag{9}$$

where

$$\theta_0 := \frac{k}{\left[k + \frac{\alpha}{1-\alpha}\right]} \mathbf{E}[\theta] \tag{10}$$

is the productivity threshold at which individuals are idle, provided that $\theta_0 \geq \underline{\theta}$. The proportion of non-working individuals in the population is equal to $F(\theta_0)$ and all of these individuals are necessarily less productive than the average. ELIE requires that $\ell(\theta) \geq k$, $\forall \theta \in \Theta$, and $\underline{\theta} \neq 0$, or, equivalently,

$$k \leq \frac{\theta}{\left[\theta + \frac{1-\alpha}{a}\right] \mathbf{E}[\theta]}, \ \forall \theta \in \Theta, \text{ and } \underline{\theta} \neq 0. \tag{11}$$

Because the RHS of the inequality in (11) is increasing in $\theta$, Proposition 1 follows.

**Proposition 1.** *With Cobb-Douglas preferences, ELIE is obtained if and only if*

$$k \leq \frac{\underline{\theta}}{\underline{\theta} + \left[\frac{1-\alpha}{\alpha}\right] \mathbf{E}[\theta]} =: k_m \text{ and } \underline{\theta} \neq 0. \tag{12}$$

By (10), there are *no idle individuals in the population* if $\theta_0 < \underline{\theta}$, i.e., if

$$k < \min\left\{1, \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\underline{\theta}}{\mathbf{E}[\theta] - \underline{\theta}}\right)\right\} =: k_0. \tag{13}$$

Both threshold values of $k$, $k_0$ and $k_m$, are useful in the subsequent analysis. Because $k_m < k_0 \leq 1$, the individual labour response to Kolm's formula imposes restrictions on the values of equalization labour $k$ that are compatible with ELIE. Because $1 - \alpha$ corresponds to the share of the full income any individual devotes to leisure, it seems reasonable to expect $\alpha$ not to exceed 0.5. If so, $k_m$ is less than 0.5. The condition that $k \leq k_m$ might thus be regarded as restrictive. In addition, if there are individuals whose ability is zero in the population (i.e., if $\underline{\theta} = 0$), an impossibility result is obtained.

## 4 Kolm's Formula and ELIE as First-Best Tax Schemes

In essence, a Kolm formula tax scheme of degree $k$ and, thus, the ELIE tax scheme of degree $k$ can be regarded as linear taxes based on productivity, with a marginal tax rate of $k$ accompanied by a lump-sum subsidy chosen so that the tax scheme is budget balanced. Indeed, by construction,

$$\mathbf{E}[T(\theta, k)] = k(\mathbf{E}[\theta] - \mathbf{E}[\theta]) = 0. \tag{14}$$

Provided that productivity is exogenous and publicly known, both tax schemes are examples of first-best tax schemes. The purpose of this section is to shed light on the social weights that generate these first-best tax schemes. We consider a social planner with a Bergson-Samuelson social welfare function. We are thus departing from the ethical framework considered by Kolm. However it is interesting to determine what kind of social welfare function the “Kolmian” social planner is trying to maximize.

With respect to Kolm’s tax formula, the polar situations in which $k = 0$ and $k = 1$ are well-known in the literature. When $k = 0$, laissez-faire is obtained. When $k = 1$, all individuals have the same full income $\mathbf{E}[\theta]$. Note that this corresponds to the outcome of the maximization of a pure utilitarian social welfare function if all individuals have the utility function defined in (6). In this case, there is a curse of the highly-skilled: the optimal indirect utility function is strictly decreasing in productivity provided that leisure is a normal good (Mirrlees, 1971, 1974). That is why it is henceforth assumed that $k \in (0, 1)$. In each step of our analysis, we first derive general results for Kolm’s tax formula and then restrict the range of feasible $k$ to apply them to ELIE.

## *4.1 Statement of the Problem*

The standard viewpoint of optimal income taxation is adopted. Individual social weights are given by the function $\lambda\colon \Theta \to \mathbb{R}_{++}$, which is assumed to be $\mathcal{C}^1$ almost everywhere. An *allocation* is a pair of functions $x\colon \Theta \to \mathbb{R}_+$ and $\ell\colon \Theta \to [0,1]$ that describes how consumption and labour supply vary with productivity. The corresponding tax schedule is the function $\tau\colon \Theta \to \mathbb{R}$ given by $\tau(\theta) = \theta\ell(\theta) - x(\theta)$. The policy-maker chooses the allocations that maximize social welfare

$$W = \int_\Theta \lambda(\theta)U(x,\ell)dF(\theta) \tag{15}$$

subject to the tax revenue constraint

$$\int_\Theta \tau(\theta)dF(\theta) \geq 0. \tag{16}$$

Because $W$ is homogeneous of degree one in $\lambda$, $\mathbf{E}[\lambda] = \int_\Theta \lambda(\theta)dF(\theta)$ can be normalized without loss of generality. It is convenient to choose

$$\mathbf{E}[\lambda] = 1. \tag{17}$$

The first-best tax scheme is solution to the following optimization programme:

**Problem 1.** Choose $x\colon \Theta \to \mathbb{R}_+$ and $\ell\colon \Theta \to [0,1]$ to maximize $W$ subject to the tax revenue constraint (16).

Letting $\gamma$ denote the Lagrange multiplier associated with the tax revenue constraint (16), Problem 1 amounts to maximizing

$$\int_\Theta \{\lambda(\theta)U(x,\ell) + \gamma[\theta\ell - x\}dF(\theta). \tag{18}$$

The question addressed in this section can thus be summarized as follows:

**Problem 2.** For $k \in (0,1)$, find $\lambda\colon \Theta \to \mathbb{R}_{++}$ such that the optimal tax scheme $\tau(\theta)$ at a solution to Problem 1 is given by Kolm's formula for this value of $k$.

## *4.2 Social Welfare Weights for Kolm's Tax Formula and ELIE: The Cobb-Douglas Case*

For simplicity, it is first assumed that the individual utility function is given by (6). Note that $\alpha \in (0,1)$ is the common taste parameter for consumption. Because Problem 1 is concave and $f > 0$, the necessary and sufficient first-order conditions for the solution to Problem 1 are:

$$x(\theta) = \frac{\alpha}{\gamma}\lambda(\theta),\ \forall\theta \in \Theta, \tag{19}$$

$$\ell(\theta) = \max\left\{0, 1 - \left(\frac{1-\alpha}{\gamma}\right)\left(\frac{\lambda(\theta)}{\theta}\right)\right\},\ \forall\theta \in \Theta. \tag{20}$$

In addition, the tax revenue constraint (16) must be binding at the social optimum. For the given $\alpha$, the first-order condition (19) shows that consumption is increasing in productivity if and only if the individual social weights are themselves increasing in productivity. By (19) and (20),

$$\theta\ell(\theta) - x(\theta) = \begin{cases} \theta - \frac{\lambda(\theta)}{\gamma} & \text{if } \lambda(\theta) \leq \frac{\gamma}{1-\alpha}\theta, \\ -\alpha\frac{\lambda(\theta)}{\gamma} & \text{if } \lambda(\theta) > \frac{\gamma}{1-\alpha}\theta. \end{cases} \tag{21}$$

The tax function $\tau$ at a solution to Problem 1 corresponds to Kolm's tax formula of degree $k$ if and only if $T(\theta,k) \equiv \theta\ell(\theta) - x(\theta)$. That is,

$$\lambda(\theta) = \begin{cases} \gamma[\theta(1-k) + k\mathbf{E}(\theta)] & \text{if } \dfrac{k\mathbf{E}(\theta)}{\left(k + \frac{\alpha}{1-\alpha}\right)} \leq \theta, \\ -\frac{\gamma}{\alpha}k[\theta - \mathbf{E}(\theta)] & \text{otherwise.} \end{cases} \tag{22}$$

Because $\mathbf{E}[\lambda] = 1$,

$$\gamma = \left[ \mathbf{E}(\theta) \Pr \left( \frac{k\mathbf{E}(\theta)}{\left(k + \frac{\alpha}{1-\alpha}\right)} \leq \theta \right) \right]^{-1}, \tag{23}$$

where the probability expression is the proportion of working individuals, which only depends on the exogenous parameters of the economy.

For our Cobb-Douglas example, the previous results allow us to characterize the social weights under which Kolm's tax formula is obtained as a first-best tax scheme for $k \in (0, 1)$.

**Proposition 2.** *Suppose that the individual utility function is given by* (6) *with* $0 < \alpha < 1$*. Then, for* $k \in (0, 1)$*, the first-best solution to Problem 1 generates a transfer scheme corresponding to Kolm's tax formula for k if and only if the social weights are given by*

$$\lambda(\theta, \alpha) = \begin{cases} \gamma[\theta(1-k) + k\mathbf{E}(\theta)] & \text{if } \dfrac{k\mathbf{E}(\theta)}{\left(k + \frac{\alpha}{1-\alpha}\right)} \leq \theta, \\ -\frac{\gamma}{\alpha} k[\theta - \mathbf{E}(\theta)] & \text{otherwise.} \end{cases} \tag{24}$$

The contribution of this proposition is to cast light on the *shape* of the distribution of the social weights that generate a tax scheme corresponding to Kolm's tax formula. Given the preference specification, it can first be noted that the social weights $\lambda(\theta)$ are (piecewise) linear in productivity. Moreover, if there are idle individuals in the population, these social weights must be $V$-shaped. Otherwise, they are strictly increasing for the whole population. This particular pattern for the social weights is in sharp contrast with what is usually considered in the optimal income tax literature. Indeed, the standard view is that the social weights are decreasing in ability; the rationale for this monotonicity restriction being based on the idea that the policy-maker is adverse to income inequality. Consequently, Proposition 2 demonstrates that Kolm's tax formula has a fundamentally different normative basis than does Mirrleesian optimal income tax theory.

The $V$-shape of the social weights with respect to productivity can be explained as follows. On the one hand, because $T(\theta, k) = k(\theta - \mathbf{E}[\theta])$, the more productive an idle individual, the smaller the transfer he receives. As a result, the consumption level is strictly decreasing in $\theta$ below $\theta_0$. By (19), this is only possible if the social weights are strictly decreasing. On the other hand, by (9), consumption increases with ability above $\theta_0$. Therefore, (19) implies that the social weights must be strictly increasing in productivity for the more productive part of the population. All individuals work at least $k$ units of time under the ELIE tax scheme of degree $k$. Hence, the probability in (23) is equal to 1. Therefore, in this case, Proposition 2 reduces to:

**Corollary 1.** *Suppose that the individual utility function is given by* (6) *with* $0 < \alpha < 1$*. Then, the first-best solution to Problem 1 generates a transfer* *scheme corresponding to ELIE for the given value of $k$ if and only if the social weights are given by*

$$\lambda(\theta) = (1-k)\frac{\theta}{\mathbf{E}[\theta]} + k. \tag{25}$$

## *4.3 Social Weights for ELIE: The General Case*

Further insights into the shape of the distribution of the social weights that generate an ELIE tax scheme are now provided for well-behaved preferences parameterized by $\alpha$. Every Pareto optimal allocation maximizes a weighted sum of utilities subject to the resource and technological constraints. The solution of such a maximization problem is assumed to be *interior* for the ELIE tax scheme of degree $k$, with every individual providing more than $k$ units of labour.

For the utility maximization programme (7), the indirect utility of a $\theta$-individual is

$$V(\theta,k) := U(\theta\ell(\theta,k) - T(\theta,k), \ell(\theta,k)), \tag{26}$$

where $\ell(\theta,k)$ denotes the labour supply of a $\theta$-individual. Let $\beta(\theta)$ be the Lagrange multiplier for the budget constraint (2) of a $\theta$-individual. The first-order condition with respect to consumption is

$$U'_x(x,\ell) = \beta(\theta). \tag{27}$$

By the envelope theorem,

$$\beta(\theta) = -\frac{\partial V(\theta,k)}{\partial T(\theta,k)}. \tag{28}$$

The social policy-maker maximizes $W$ subject to the tax revenue constraint (16). Recall that $\gamma$ is the Lagrange multiplier of this constraint. Thus, the first-order condition with respect to consumption is

$$U'_x(x,\ell) = \frac{\gamma}{\lambda(\theta)}. \tag{29}$$

Combining (27), (28), and (29), one obtains

$$\begin{aligned}\lambda(\theta) &= \gamma\left(-\frac{\partial V(\theta,k)}{\partial T(\theta,k)}\right)^{-1} \\ &= \frac{\gamma}{U'_x(\theta(\ell(\theta,k)-k) + k\mathbf{E}[\theta], \ell(\theta,k))}.\end{aligned} \tag{30}$$

This equation states that the weight of the utility of a $\theta$-individual in the social objective function equals the reciprocal of his marginal utility of wealth evaluated at the support prices $(1,\theta)$ and imputed lump-sum wealth $kE[\theta]$. Consequently,

$$\lambda'(\theta) = -\frac{\gamma(U'_x)^2}{\left[\left(\theta\frac{d\ell(\theta,k)}{d\theta} + \ell(\theta,k) - k\right)U''_{xx} + \frac{d\ell(\theta,k)}{d\theta}U''_{x\ell}\right]}, \tag{31}$$

where all derivatives are evaluated at $(\theta(\ell(\theta,k)-k)+k\mathbf{E}[\theta],\ell(\theta,k))$.

Using the Slutsky equation,

$$\frac{d\ell(\theta,k)}{d\theta} = \frac{\partial h(\theta,k)}{\partial\theta} - (\ell(\theta,k)-k)\frac{\partial\ell(\theta,k)}{\partial T(\theta,k)}, \tag{32}$$

where $h(\theta,k)$ is the Hicksian labour supply. By the law of compensated demand, the first term of the Slutsky equation—which captures the substitution effect of changing $\theta$—is positive. Because $\ell(\theta,k)-k\geq 0$, the second term—which captures the income effect—is non-positive provided that leisure is a normal good, i.e., if $\partial\ell(\theta,k)/\partial T(\theta,k)>0$. Recalling that $U''_{xx}<0$ and that there is *ALEP complementarity* between consumption and labour (or *ALEP substitutability* between consumption and leisure) if and only if $U''_{x\ell}>0$ (see Allen, 1934; Auspitz and Lieben, 1889; Edgeworth, 1925; Pareto, 1906), Proposition 3 follows from (31) and (32).

**Proposition 3.** *Assume* (i) *that there is ALEP substitutability between consumption and leisure and* (ii) *that the substitution effect on labour supply is larger than the income effect. Then, the social weights that generate ELIE are strictly increasing in productivity.*

## 5 Implementation of ELIE

This section focuses on the implementability of the ELIE tax schemes. Following Mirrlees (1971), we assume that productivity differences are the only source of heterogeneity.

### *5.1 The Implementation Issue*

Mirrlees (1971, p. 208) notes in the conclusion of his seminal article on optimal income taxation that "it would be good to devise taxes complementarity to the income tax, designed to avoid the difficulties that the tax is faced with ... [T]his could be achieved by introducing a tax schedule that depends upon time worked as well as upon labour-income." In fact, once the government knows both variables in the Mirrlees framework, it is capable of inferring the productivity level of each individual because gross income is the product of productivity and time worked. Thus, there seems to be no reason not to design a tax based on individual skills. Accordingly, Kolm (2004, p. 175) considers that "scholars who let their thinking be directed by casual superficial remarks about difficulties of implementation and in particular information are bound to run in the wrong direction."

There are however different arguments that temper the idea that there has been a misplaced emphasis on implementation. The starting point is to distinguish occupations according to whether or not the labour time is verifiable by an employer.

On the one hand, there are jobs for which an employer can precisely record the hours that people work, thanks to a time clock for instance. In this case, the labour time $\ell(\theta)$ can be obtained by the policy-maker, possibly at some cost. The extraction of this information belongs to the economics of tax evasion (Cowell, 1990). However, it would be incorrect to conclude from the fact that the labour supply of a $\theta$-individual is observable that the ratio of his gross income to the number of hours worked necessarily is his correct productivity level $\theta$. This point has been made clear by Dasgupta and Hammond (1980). Any $\theta$-individual has the possibility to provide labour at a lower productivity level $\theta'$. In other words, $\theta$ is the maximum productivity level of a $\theta$-individual. Formally, let $\ell(\theta';\theta)$ be the labour supply of a $\theta$-individual at productivity $\theta'$. Then, his total labour supply is $\int_{\underline{\theta}}^{\theta} \ell(\theta';\theta)d\theta'$. The computed productivity $\theta^c$ is obtained as the ratio of his total gross income $\int_{\underline{\theta}}^{\theta} \ell(\theta';\theta)\theta' d\theta'$ to his total labour supply, i.e., $\theta^c = \int_{\underline{\theta}}^{\theta} \ell(\theta';\theta)\theta' d\theta' / \int_{\underline{\theta}}^{\theta} \ell(\theta';\theta)d\theta'$. It is equal to $\theta$ if and only the tax scheme gives a $\theta$-individual an incentive to provide all labour using his actual skill so that his labour supply is $\ell(\theta;\theta)$. In all other cases, taxing the computed productivity can no longer be considered as a lump-sum tax because the computed productivity is endogenous to the tax scheme. In other words, it is incorrect to conclude from the absence of cheating on time worked that the best productivity level of an individual is public knowledge. *For this type of labour, work-time evasion and implementation are both meaningful issues.*

On the other hand, there are other occupations for which it proves difficult to separate time worked from leisure. This observation notably applies to individuals involved in "brainwork" occupations for which the use of a time clock would be completely irrelevant. Because the time spent working is not verifiable, nobody can establish that a given individual in this kind of occupation is cheating. So, *the problem of work-time evasion does not exist for this second type of labour, whilst the implementation issue must still be taken into account.* This last issue is particularly important because the individuals involved in these kinds of occupations are likely to belong to the more productive part of the population and to pay positive taxes.

Of course, in a more general framework in which productivity is the product of innate talent and effort, the knowledge of time worked would not be sufficient to identify productivity either. We deliberately place ourselves in the simplest case, where effort is not taken into account, to examine if ELIE is incentive compatible.

## 5.2 Incentive Compatibility of ELIE

Incentive compatibility of ELIE basically depends on the verifiability of gross income and time worked, i.e., on *which variables can be included in the contract between the policy-maker and an individual worker*. By definition, a variable is verifiable if a contract that depends on it can be enforced by a third party (e.g., an arbitrator or court) who can verify the value of the variable and make the parties fulfill the contract. Hence, a contract can only depend on verifiable variables. This subsection clarifies key points regarding incentive-compatibility; its contribution is mainly pedagogical.

Let us consider individuals who are faced with the ELIE tax scheme of degree $k$ ($k \neq 0$). Three cases need to be distinguished.

### 5.2.1 Non-Verifiability of Gross Income and Hours Worked

In the first case, neither gross income $z$ nor time worked $\ell$ are verifiable. Thus, the government has no means of recovering the true productivity level of an individual from knowledge of $z$ and $\ell$. Consequently, *the tax base is purely declaratory*. Every individual thus has an incentive to claim that his gross income $z$ and his labour time $\ell$ are such that $z/\ell = \underline{\theta}$. Indeed, he then obtains the maximum transfer $-T(\underline{\theta}, k)$, which maximally relaxes his budget constraint. As a consequence,

$$\int_{\Theta} T(\underline{\theta}, k) dF(\theta) = k(\underline{\theta} - \mathbf{E}[\theta]) < 0, \tag{33}$$

which implies that the tax revenue constraint (2) is necessarily violated. Therefore, in this case, *the ELIE tax scheme is not incentive compatible.*

### 5.2.2 Verifiability of Gross Income and Hours Worked

The polar case is now considered: gross income and hours worked are verifiable and included in the contract between the policy-maker and the taxpayer. In this case, it is known from Dasgupta and Hammond (1980) that a tax schedule is incentive-compatible if indirect utility is non-decreasing in productivity.

To gain further insight into this finding, it is worth describing the timing of the game between the taxpayers and the policy-maker. Let $\hat{\theta}$ be the productivity level at which labour is actually supplied.

1. First, the tax authority announces a tax schedule
$$T(\hat{\theta}, k) = k(\hat{\theta} - \mathbf{E}[\hat{\theta}]). \tag{34}$$
2. Second, for all $\theta \in \Theta$, each $\theta$-individual chooses at which productivity level $\hat{\theta} = z/\ell$ (with $\hat{\theta} \le \theta$) he wants to provide his labour. He then pays a common contribution $k\hat{\theta}$ and receives $k\mathbf{E}[\hat{\theta}]$ from society.

As is standard in optimal income taxation theory, we are interested in implementation in weakly dominant strategies. In this context, the tax schedule (34) is implementable if each individual cannot increase his well-being by hiding his true productivity level $\theta$.

For convenience, it is assumed that a $\theta$-individual can only provide labour at *one* skill level $\hat{\theta} \le \theta$. When his gross income is $z$ and his time worked is $\ell$, his observed skill is given by $\hat{\theta} = z/\ell$. Therefore, his utility level when he chooses $z/\ell = \hat{\theta}$ is given by

$$\max_{\ell_{\min}(\hat{\theta}) \le \ell \le 1} U(\hat{\theta}\ell - T(\hat{\theta}, k), \ell) =: V(\hat{\theta}), \tag{35}$$

where $V(\hat{\theta})$ is the indirect utility of a $\hat{\theta}$-individual. Hence, a $\theta$-individual chooses to work with the skill level $\hat{\theta}$ that solves

$$\max_{\hat{\theta} \le \theta} V(\hat{\theta}). \tag{36}$$

Accordingly, *every individual maximizes his utility when working at his best skill if* $V(\hat{\theta})$ *is non-decreasing in productivity.* Applying the envelope theorem to
$$V(\hat{\theta}) \equiv U(\hat{\theta}(\ell(\hat{\theta}) - k) + k\mathbf{E}[\hat{\theta}], \ell(\hat{\theta})), \tag{37}$$
one obtains

$$V'(\hat{\theta}) = U'_x(\hat{\theta}(\ell(\hat{\theta}) - k) + k\mathbf{E}[\hat{\theta}], \ell(\hat{\theta}))[\ell(\hat{\theta}) - k]. \tag{38}$$

Hence, because $U'_x > 0$, the indirect utility $V(\hat{\theta})$ is non-decreasing in $\hat{\theta}$ if and only if
$$\ell(\hat{\theta}) \ge k, \tag{39}$$
which is satisfied with the ELIE tax scheme of degree $k$. In summary:

**Proposition 4.** *The ELIE tax scheme of degree $k$ is implementable as a direct truthful mechanism in weakly dominant strategies if both gross income and time worked are observable by the policy-maker and verifiable by a third party.*

In this sense "the individuals choose to work with their best skills and thus to 'reveal' their capacities and to exhibit their economic value." (Kolm, 2010, p. 118, emphasis omitted) Hence, *ELIE is incentive compatible for all individuals in occupations for which a "time clock" can be used.*

### 5.2.3 Verifiability of Gross Income and Non-Verifiability of Hours Worked

It remains to examine if the preceding result extends to the numerous individuals working in occupations for which the use of a time clock is irrelevant. In this case, time worked is no longer verifiable, so that it would be useless to include it in the contract between the policy-maker and a taxpayer. Hence, a $\theta$-individual no longer needs to spend the same time working as a $\hat{\theta}$-individual if he chooses to earn the same gross income as the latter.

A natural solution in this case is to refer to some legal definition of the time worked, like the average working time $\overline{\ell}$, for example. The policy-maker infers that the productivity of a $\theta$-individual with gross income $z$ is $z/\overline{\ell}$. If this ratio is used as a tax base, then a Kolm formula tax scheme of degree $k$ becomes

$$T\left(\frac{z}{\overline{\ell}},k\right)=k\left(\frac{z}{\overline{\ell}}-\mathbf{E}\left[z/\overline{\ell}\right]\right)=\frac{k}{\overline{\ell}}(z-\mathbf{E}[z]). \tag{40}$$

The tax function in (40) is a linear tax on gross income, with marginal tax rate $k/\overline{\ell}$ and basic income $k\mathbf{E}\left[z/\overline{\ell}\right]$. It is budget balanced because, by construction, $\mathbf{E}\left[T\left(z/\overline{\ell},k\right)\right]=0$. Moreover, it is incentive compatible if $k/\overline{\ell}<1$. However, although this function is linear in gross income, it does not correspond to a first-best ELIE tax scheme, which is linear in labour productivity.

An alternative solution to the implementation problem is to ask every individual to report his non-verifiable time worked $\hat{\ell}$ in conjunction with his verifiable gross income $\hat{z}$. The inferred productivity level $\hat{\theta}=\hat{z}/\hat{\ell}$ is then used as a tax base. The hidden productivity level is $\theta=\hat{z}/\ell$, where $\ell$ is the actual time worked. In this case, a $\theta$-individual chooses

$$\left(\hat{z}^{*}\left(\theta\right),\hat{\ell}^{*}\left(\theta\right)\right)=\underset{\substack{\hat{z}-T\left(\frac{\hat{z}}{\hat{\ell}},k\right)\geq 0,\\ 0\leq\hat{\ell}\leq 1}}{\arg\max}\;U\left(\hat{z}-T\left(\frac{\hat{z}}{\hat{\ell}},k\right),\frac{\hat{z}}{\theta}\right). \tag{41}$$

Hence, every $\theta$-individual claims that he works $\hat{\ell}^{*}(\theta)=1$ in order to maximize his utility provided that $T$ is increasing in gross income. Because he never chooses to work 24 hours a day, it follows that he *overstates* his labour time in such a way that the policy-maker *underestimates* his productivity. That is, he chooses $\hat{\theta}$ so that $\hat{\theta}=\hat{z}^{*}(\theta)<\theta$.

**Proposition 5.** *Suppose that gross income is verifiable and that time worked is non-verifiable. Then, if the ELIE tax scheme of degree $k$ is based on the* *inferred productivity level $\hat{\theta}$, everybody has an incentive to overstate his time worked so as to understate his true productivity level $\theta$. Therefore, this tax scheme is not implementable as a direct truthful mechanism in weakly dominant strategies.*

Recall from Proposition 2 that in order for an ELIE tax scheme to be optimal when there is no private information, the social weights must be strictly increasing in productivity. However, even with this restriction on the weights, Proposition 5 demonstrates that ELIE is not sufficiently favourable to the highly-skilled individuals to prevent every individual from *understating* his productivity level when hours worked are non-verifiable.

## 5.3 Implications

In practice, there are many occupations in which labour time is not verifiable by the employer. Moreover, individuals in these occupations are on average highly skilled and, by Proposition 5, understate their true productivity level when we depart from the first-best setting. Thus, there is a difficulty in implementing ELIE when hours worked are non-verifiable.

There are different routes to construct a second-best alternative to the ELIE tax scheme of degree $k$. A first route would retain the social weights (24) that generate Kolm's tax formula as a solution to Problem 1 and use them to solve the second-best problem which takes incentive-compatibility into account. However, there seems to be no reason why a welfarist objective that coincides with Kolm's scheme in the first-best context would be relevant to Kolm's philosophy in the second-best. A second route, which is followed in this article and that does not rest on welfarism, aims at implementing the *first-best* transfers of ELIE in a second-best setting.

For illustrative purposes, it is henceforth assumed that the population consists of two types of individuals who are involved in brainwork occupations, with respective productivities $\underline{\theta}$ and $\overline{\theta}$ ($\underline{\theta} < \overline{\theta}$).

### 5.3.1 A Second-Best Alternative to ELIE: Principles

Given the usual results of contract theory (see, e.g., Laffont and Martimort, 2002), one expects that when a second-best alternative to ELIE is implemented that (i) the optimal tax schedule will not involve any distortion of the labour supply of the high-type individual and (ii) the only binding incentive-compatibility constraint will be that of the high-type individual. Therefore, in a two-type population, the basic idea is to examine to what extent the utility of the low type must be decreased to make the high type indifferent between his own bundle and that of the low type. For this purpose, it is sufficient to introduce a linear tax, faced by the low type $\underline{\theta}$, that distorts his labour supply and that ensures budget balancedness through an increase in the lump-sum subsidy to this type. For convenience, it is assumed that there are only two individuals in the population, one for each ability level. The construction of a budget-balanced and incentive-compatible allocation proceeds in four steps.

First, when the low-skilled individual faces a distortionary tax rate $t$ on gross income and a second-best lump-sum transfer $\widetilde{T}(\underline{\theta}, k)$, his budget constraint is

$$x(\underline{\theta}) = (1-t)z(\underline{\theta}) - \widetilde{T}(\underline{\theta}, k). \tag{42}$$

The first-order condition for the utility maximization programme of the low type states that his marginal rate of substitution at the optimal bundle is equal to his net-of-tax wage rate, i.e.,

$$-\frac{U_z'(x(\underline{\theta}), z(\underline{\theta})/\underline{\theta})}{U_x'(x(\underline{\theta}), z(\underline{\theta})/\underline{\theta})} = 1 - t. \tag{43}$$

Second, the high type has no incentive to mimic the low type if the utility the former obtains at his own bundle $\left(x\left(\overline{\theta}\right), z\left(\overline{\theta}\right)\right)$ is not less than what he receives from the bundle $(x(\underline{\theta}), z(\underline{\theta}))$ of the latter, i.e., if $V\left(\overline{\theta}; \overline{\theta}\right) \geq V\left(\underline{\theta}; \overline{\theta}\right)$, where

$$V(\theta', \theta) := U\left(x(\theta'), \frac{z(\theta')}{\theta}\right). \tag{44}$$

In order to minimize the loss in efficiency, this inequality must be binding at the optimum. Therefore, the second-best allocation must satisfy

$$V\left(\overline{\theta}; \overline{\theta}\right) = V\left(\underline{\theta}; \overline{\theta}\right). \tag{45}$$

Third, the net transfer to the low-skilled individual must be the same as in the first best $T(\underline{\theta}, k)$, which ensures that the high-skilled individual pays the same tax as in the first best. Formally,

$$\widetilde{T}(\underline{\theta}, k) + tz(\underline{\theta}) = T(\underline{\theta}, k). \tag{46}$$

As a consequence,

$$\widetilde{T}(\underline{\theta}, k) < T(\underline{\theta}, k), \tag{47}$$

implying that the second-best lump-sum transfer $-\widetilde{T}(\underline{\theta}, k)$ must be greater than the first best transfer $-T(\underline{\theta}, k)$.

The fourth step consists in checking that the low type has no incentive to mimic the high type, i.e., that

$$V(\underline{\theta}; \underline{\theta}) > V\left(\overline{\theta}; \underline{\theta}\right). \tag{48}$$

The second-best problem is therefore:

**Problem 3.** Choose $\widetilde{T}(\underline{\theta}, k)$ and $t \in (0, 1)$ such that (42), (43), (45), (46), and (48) are satisfied.

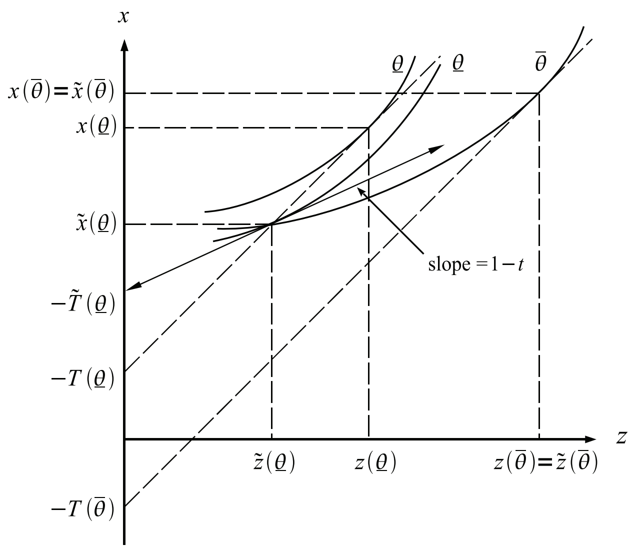


**Fig. 4** Geometric construction

Figure 4 shows the logic of the construction of the solution to Problem 3. (In this figure, $k$ is suppressed from the notation; $x$, $z$, and $T$ correspond to first-best levels; $\tilde{x}$, $\tilde{z}$, and $\widetilde{T}$ to second-best levels; and the indifference curves are labelled by productivity levels.) The two budget lines for the low-skill individual cross at the new equilibrium for this individual. A simple example is now provided to show the practicality of this approach.

### 5.3.2 A Second-Best Alternative to ELIE: An Example

A complete solution to Problem 3 is now provided for the case in which both individuals have the same Cobb-Douglas utility function

$$U(x, \ell) = x^{\alpha}(1-\ell)^{1-\alpha}. \tag{49}$$

For illustrative purposes, it is assumed that $k = 1/4$, $\alpha = 1/2$, $\underline{\theta} = 1$, and $\overline{\theta} = 3/2$.

In the first-best setting, the ELIE tax scheme of degree $k$ is such that

$$T(\underline{\theta}, k) = \frac{1}{4}\left(1 - \frac{1}{2}\left(1 + \frac{3}{2}\right)\right) = -\frac{1}{16} \tag{50}$$

**Table 1** First-best and second-best allocations corresponding to ELIE

| First-best | | | | | |
|---|---|---|---|---|---|
| $\theta$ | $x$ | $\ell$ | $z$ | $V(\underline{\theta};\theta)$ | $V(\overline{\theta};\theta)$ |
| $\underline{\theta}$ | 0.531 | 0.469 | 0.469 | 0.531 | 0.397 |
| $\overline{\theta}$ | 0.719 | 0.521 | 0.781 | 0.604 | 0.587 |
| Second-best | | | | | |
| $\theta$ | $x$ | $\ell$ | $z$ | $V(\underline{\theta};\theta)$ | $V(\overline{\theta};\theta)$ |
| $\underline{\theta}$ | 0.475 | 0.413 | 0.413 | 0.528 | 0.397 |
| $\overline{\theta}$ | 0.719 | 0.521 | 0.781 | 0.587 | 0.587 |

and $T\left(\overline{\theta},k\right)=1/16$. Because $k<k_m\simeq 0.44$, it is known from Proposition 1 that every individual provides labour in excess of the common contribution $k$. Table 1 gives the corresponding first-best labour supply, consumption, and utility levels. Although $V(\underline{\theta};\underline{\theta})<V\left(\overline{\theta};\overline{\theta}\right)$ at the first-best solution, it can be verified that the high-skilled individual has an incentive to misreport his productivity, while the low-skilled individual reveals his type truthfully. The mimicking behaviour of the high-skilled individual results in a public deficit equal to $T(\underline{\theta},k)\times 2=1/8$. The budget-balance constraint is thus violated. This observation illustrates the result obtained in Proposition 5: *the high-skilled individual does not choose to work with his best possible skill thereby "revealing" his capacity and exhibiting his economic value*.

The second-best Problem 3 amounts to finding $t$ and $T(\underline{\theta},k)$ that solve the following system of four equations before checking that the low-skilled individual has no incentive to overstate his productivity level:

$$V\left(\overline{\theta};\overline{\theta}\right)=x^{\alpha}(\underline{\theta})\left(1-\frac{\underline{\theta}\ell(\underline{\theta})}{\overline{\theta}}\right)^{1-\alpha}, \tag{51}$$

$$\left[\frac{1-\alpha}{\alpha}\right]\left[\frac{x(\underline{\theta})}{1-\ell(\underline{\theta})}\right]=(1-t)\underline{\theta}, \tag{52}$$

$$x(\underline{\theta})=(1-t)\underline{\theta}\ell(\underline{\theta})-\widetilde{T}(\underline{\theta},k), \tag{53}$$

$$\widetilde{T}(\underline{\theta},k)+t\underline{\theta}\ell(\underline{\theta})=T(\underline{\theta},k), \tag{54}$$

where $x\left(\overline{\theta}\right)\simeq 0.719$, $\ell\left(\overline{\theta}\right)\simeq 0.521$, and $T(\underline{\theta})=-1/16$. Using (52), (53), and (54)), one obtains

$$x(\underline{\theta})=\alpha\left[\frac{1-t}{1-\alpha t}\right](\underline{\theta}-T(\underline{\theta},k)), \tag{55}$$

$$\ell(\underline{\theta}) = \frac{1}{1-\alpha t}\left[\alpha(1-t) + \left(\frac{1-\alpha}{\underline{\theta}}\right) T(\underline{\theta}, k)\right], \tag{56}$$

which are substituted in (51) to get

$$V\left(\overline{\theta}; \overline{\theta}\right) = \left[\alpha\left(\frac{1-t}{1-\alpha t}\right)\left(\underline{\theta} - T(\underline{\theta}, k)\right)\right]^{\alpha} \times \left(1 - \frac{\alpha\underline{\theta}(1-t) + (1-\alpha)T(\underline{\theta}, k)}{\overline{\theta}(1-\alpha t)}\right)^{1-\alpha}. \tag{57}$$

It then remains to solve (57) for $t$ and substitute the obtained value in (55) and (56). It is found that $t = 19.12\%$ and, hence, that $\widetilde{T}(\underline{\theta}, k) = -0.1414$ instead of $T(\underline{\theta}, k) = -0.0625$. Table 1 provides the other second-best results. As expected, the low-skilled individual has no incentive to mimic the high-skilled one, which confirms that it was not necessary to take his incentive-compatibility constraint (48) explicitly into account. In addition, his second-best indirect utility is only slightly reduced compared to the first-best one ($\simeq -0.56\%$). This is a very modest price to pay for the ELIE tax scheme to induce individual truth-telling.

The basic idea is to worsen the position of the low-skilled individual compared to the first-best allocation in such a way that this worsening is seen to be more painful to the high-skilled individual than to the low-skilled individual. It is possible to do this because the two types of individuals do not value a reduction in gross income in the same way. The adjustment proceeds as follows. First, the introduction of the distorting tax gives rise to a substitution effect that induces the low-skilled individual to reduce his labour supply. He is impoverished by the associated income effect and is thus encouraged to work more. In general, the variation in labour supply is ambiguous, whereas consumption is reduced. However, in the example, the substitution effect prevails because the income effect is partially compensated for by the increase in the lump-sum transfer to the low-skilled individual. Thus, the low-skilled individual chooses to increase his leisure to the detriment of consumption. Second, both individuals value the reduction in consumption in the same way because they have the same utility function. However, they do not equally value the impact of the decrease in gross income required from the low-type individual. This reduction translates into a smaller increase in leisure for the high-skilled individual than for the low-skilled individual because the former is more productive than the latter.

## 6 Concluding Comments

ELIE has some very attractive and striking features. It corresponds to the idea that laissez-faire should be implemented over a certain threshold of labour time, with the income from this equal labour being given by everyone to share among all citizens. However, it must be conceded that the ELIE tax scheme derived by Kolm is not without its shortcomings.

First, its definition involves an endogeneity condition whose satisfaction requires some qualifications. Second, it subjects the more productive part of the population to "corvée labour" and embodies, therefore, some troubling "feudalistic" features. Third, in the first-best framework and under weak assumptions, it is generated by social weights that are strictly increasing in productivity for all working individuals. If Kolm's proposal should get credit for expanding the set of ethically acceptable social weights, it could nevertheless encounter stiff opposition from some normative economists because of the gross income and consumption profiles it is associated with. Fourth, it is only implementable as a truthful mechanism in weakly dominant strategies when both gross income and time worked are observable and verifiable. Because we believe that non-verifiability of time worked is a key feature of brainwork occupations, it seems to us that the implementation issue must be fully addressed. A simple method has been proposed when there are only two skill levels so as to obtain the first-best transfers of ELIE in a second-best world in which time worked is not verifiable. This method constitutes the first step of a more complete investigation of the implementation issue for a larger number of types that is carried out in a companion article (see Simula and Trannoy, 2010).

More generally, it seems to us that the endogeneity condition included in the definition of ELIE raises difficulties. For instance, it is argued by Kolm that, under ELIE, individuals benefit from a basic income $k\mathbf{E}[\theta]$ (see Kolm, 2010, p. 111). Because a basic income is by definition the income received by an individual who is not working, this assertion seems problematic, at least for the more productive part of the population. For this reason, it may be promising to study an income tax schedule that combines a flat tax at rate $\tau$ with an exemption for overtime labor. If $m$ denotes the basic income, this "overtime exemption flat tax schedule" is defined by setting

$$T(z,\ell;\tau,k) = \tau \min\left\{z, k\frac{z}{\ell}\right\} - m. \tag{58}$$

This tax schedule allows us to distinguish two regimes depending on whether an individual has earnings below or above a threshold income. This threshold corresponds to the earnings of an individual who works $k$ hours at his apparent productivity $z/\ell$. It is obviously supposed that both gross income and time worked are verifiable. When the labour income is smaller than this level, the tax formula reduces to a regular flat tax; when the labour income is higher, the tax liability is capped by the threshold, as shown in Figure 5. This overtime exemption flat tax is incentive compatible provided that $\tau$ is less than 1. A more thorough study of such a tax schedule is left for future research.

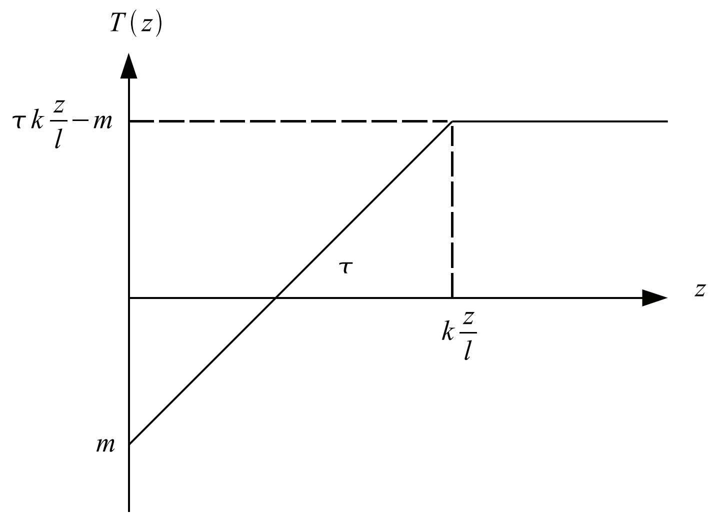


**Fig. 5** Overtime exemption flat tax

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