Chapter 7
An Exploration of Incentive-Compatible ELIE

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Abstract Simula and Trannoy (2011) have shown that ELIE is confronted with implementation issues when the policymaker cannot observe the time worked by every individual. This paper tries to fix this problem. To this aim, we characterise the second-best allocations which are the closest to ELIE first in terms of welfare and then in terms of transfers. In the former perspective, we consider a welfarist setting in which the social weights are those required by ELIE to be generated as a first-best allocation. These weights are defined by the tangent hyperplane to the first-best Pareto set at the ELIE allocation. We show that, in the absence of income effect on labour supply, the closest solution to ELIE is the *laissez-faire*. In addition, simulations for a Cobb–Douglas economy show that the second-best transfers may then be substantially different from ELIE. This is why, in the latter perspective, we construct second-best allocations which are both incentive-compatible and generate net transfers coinciding with the first-best ELIE transfers. We show that the unique solution is Pareto-efficient in the constraint set.

7.1 Introduction

In Mirrlees (1971) seminal article, the policy-maker aims at redistributing income from the high to the low productive individuals. However, it is confronted with a basic separation between public and private information: "the

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Very helpful comments from François Maniquet are gratefully acknowledged. The usual caveat applies.

C. Gamel and M. Lubrano (eds.), *On Kolm's Theory of Macrojustice*,
DOI 10.1007/978-3-540-78377-0_7,

government can observe the total product of each individual, that is the product of the wage rate and the amount worked, but is unable to observe either of these alone" (Mirrlees 1997). As a result, the tax scheme must be based on gross income instead of skill levels. However, as emphasised by Mirrlees himself in the conclusion of his paper, "it would be good to devise taxes complementary to the income-tax, designed to avoid the difficulties that tax is faced with".

The tax scheme recently proposed by Serge-Christophe Kolm belongs to this family (Kolm 2005). It is derived from fundamental principles of justice and corresponds in essence to a linear tax based on productivity. It has a remarkable feature when every individual to whom it applies provides a quantity of labour in excess to what is needed for him to pay his net tax. In this case, everyone gives the fruit of the same labour duration to the society from which he receives the average donation. This structure is referred to as *"Equal-Labour Income Equalisation"* or ELIE by Kolm and constitutes a simple way of equally apportioning the common contribution between the citizens, according to their means: "from each, to each other, the product of the same labour" and "from each, to each other, according to her capacities".

It is argued by Kolm (2007, unpublished manuscript) that ELIE is incentive-compatible in the sense that "it induces people to work with their full abilities". In this respect, ELIE would resolve the fundamental trade-off between equity and efficiency and justify leaving the second-best analysis for the first-best one. In fact, the incentive-compatibility of ELIE depends on which variables are observable and can thus be included in the contract between the taxpayers and the policymaker. If it seems natural to consider that gross income is observable, the verifiability of time worked appears to be more problematic. In practice, when both gross income and time duration can be observed in a second-best world, ELIE actually provides the right incentives for every agent whose labour can be time-clocked or measured by similar mechanisms. However, this is not always the case for all individuals in brainwork occupations for which time and attendance solutions for employee labour tracking are irrelevant. In Simula and Trannoy (2011), it is established – in a continuous population framework – that these individuals have an incentive to misreport their productivity through the gross-income/labour combination they choose as soon as time worked is not observable and verifiable. This is an unfortunate state of affairs because the latter individuals are likely to be the most productive in the population, i.e., those who are supposed to pay the higher taxes under ELIE. Consequently, incentive-compatibility of ELIE is not a trivial matter.

This paper considers a discrete population version of the framework developed by Mirrlees (1971), initiated by Stiglitz (1982) and Guesnerie and Seade (1982), and focuses on the cases in which the first-best ELIE allocation is not envy-free. Consequently, ELIE is not implementable when productivity is not observable by the policymaker. ELIE must thus be replaced by an income tax scheme. Our primary objective is to construct income tax schemes which are "as close as possible" to ELIE. Proximity is defined is two ways. First, we keep the social weights which generate ELIE in the first-best setting to define the second-best social objective. In this sense, this approach yields the closest solution in terms of welfare. We then maximise this social objective function to find the best income tax scheme in the possibility set constrained both by the incentive-compatibility constraints and the tax revenue constraint. We show that the income tax can be substantially different from ELIE. In particular, in the absence of income effect on labour supply, i.e., for quasilinear-in-consumption preferences, the closest solution to ELIE is the laissez-faire. Second, because Kolm analysis puts the stress on the specific shape of the ELIE transfers, we endeavour to stay as close as possible to ELIE transfers. To this aim, we depart from the welfarist framework to adopt a non-welfarist viewpoint as favoured by Kolm in *Macrojustice* (Kolm 2005). We look for the income tax scheme for which everyone pays the same tax or receives the same transfer as in the first-best while the second-best incentive-compatibility constraints are satisfied. We show that there is a unique solution which is Pareto-efficient and corresponds to a simple monotonic chain to the left, i.e., to an allocation of gross-income/consumption bundles in which every individual is indifferent between his own bundle and that of his nearest less productive neighbour. The general methodology to get the closest solution in terms of transfers is then illustrated through an example.

A related analysis is made by Fleurbaey and Maniquet (2011) who also address the incentive-compatibility of ELIE in both above-mentioned informational settings. A major difference comes from the diversity of preferences over leisure they introduce. Kolm regards this diversity as a private matter and not as a legitimate ground for redistribution, contrary to productivity differences. Therefore, from an ethical viewpoint, it seems justified – as a first pass – to focus on the latter and to consider the restrictive framework where all individuals have the same preferences. To cope with this additional heterogeneity, Fleurbaey and Maniquet employ a different social welfare function from utilitarianism, which is less demanding in terms of interpersonal utility comparisons. An axiomatic foundation of ELIE is provided in this context.

The paper is organised as follows. Section 7.2 addresses the incentive-compatibility of ELIE. Section 7.3 examines the closest solution in terms of welfare. Section 7.4 is devoted to the closest solution in terms of transfers. Section 7.5 provides concluding comments.

7.2 ELIE and Incentive-Compatibility

7.2.1 *Setting*

We consider a competitive economy in which the technology exhibits constant returns to scale. The population consists of $I \geq 2$ individuals, indexed by $i \in \mathscr{I} := \{1, \ldots, I\}$. For convenience, only one person has a given productivity level. This simplification is not particularly restrictive because the distance between two productivity levels is free to vary. Without loss of generality, the vector of productivities $w := (w_1, \ldots, w_I)$ is taken to be monotonically increasing, belonging to the set

$$\Omega := \left\{ w \in \mathbb{R}^I_{++} | w_1 < \cdots < w_I \right\}. \tag{7.1}$$

Person i's wage rate is fixed, equal to his productivity w_i. The time endowment of each individual has been normalised to equal 1. Consumption good is chosen as the *numéraire*. When person i works ℓ_i units of time, his gross income amounts to

$$z_i := w_i \ell_i, \qquad i \in \mathscr{I}. \tag{7.2}$$

All individuals have the same preferences over consumption and leisure. These preferences are represented by a cardinal utility function $U : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}$. The function U is assumed to be strictly concave, twice-continuously differentiable, strictly increasing in consumption and strictly decreasing in labour. Moreover, leisure is assumed to be a normal good. Note that person i has a personalised utility function in the gross-income/consumption space, defined by $u : \mathbb{R}_+ \times [0, w_I] \to \mathbb{R}$ with

$$u (x, z; w_i) := U (x, z/w_i). \tag{7.3}$$

Person i's marginal rate of substitution of gross income for consumption at the (x_j, z_j)-bundle is defined as

$$s (x_j, z_j; w_i) := -\frac{u'_z (x_j, z_j; w_i)}{u'_x (x_j, z_j; w_i)}. \tag{7.4}$$

The Spence–Mirrlees condition is assumed to be satisfied. It ensures that, for any given gross-income/consumption bundle, more productive individuals have flatter indifference curves. Formally:

Assumption 7.1 *Given any* $(x_k, z_k) \in \mathbb{R}_+ \times [0, w_I]$,

$$s (x_k, z_k; w_i) > s (x_k, z_k; w_j) \Leftrightarrow i < j. \tag{7.5}$$

7.2.2 ELIE

The tax policy can now be introduced formally. With a *Kolm formula tax scheme of degree* k, person i is required to transfer kw_i to society in exchange for which he receives $k\mathbb{E}(w)$, where $\mathbb{E}(w)$ is the average productivity level in the population. Hence, the tax function is

$$T_i(k) = k(w_i - \mathbb{E}(w)). \tag{7.6}$$

The *ELIE tax scheme of degree* k combines (a) the Kolm formula tax scheme of degree k with (b) a condition on the endogenous individual labour supply. In the absence of involuntary unemployment, this condition states that all productive individuals must provide labour in excess of k to take part in the overall redistributive mechanism (Kolm 2007, pp. 26–27, unpublished manuscript). Because it can be argued that a redistributive tax schedule should be universal with the same schedule applied to everyone, special attention is paid below to the case in which all citizens face the ELIE tax scheme of degree k.[1]

7.2.3 ELIE, Envy and Implementability

In the first-best setting, the policymaker observes the productivity of every individual. He implements the tax scheme $T_i(k)$ based on individual skill levels. We say that person i envies person j if and only if, given his own productivity, the former would prefer the labour/consumption bundle of the latter, i.e., if and only if

$$U\left(x_j, \frac{z_j}{w_i}\right) > U\left(x_i, \frac{z_i}{w_i}\right). \tag{7.7}$$

Figure 7.1 shows the budget lines and the choices of two individuals, with $w_2 > w_1$: person 2 may envy person 1 at the ELIE allocation because of the progressivity of the tax schedule $T_i(k)$, but the converse is impossible. More generally, when there are more than two individuals, person i may envy person j at the ELIE allocation only if $w_i > w_j$; but the contrary is impossible. Figure 7.2 shows a situation where person 2 does not envy person 1 at the

[1] These desiderata are incorporated in the actual tax schedules in many developed countries. In France, for instance, the 13th article of the Declaration of the Rights of Man and of the Citizen, which has a constitutional status, states that the common contribution should be equitably distributed among *all* the citizens in proportion to their means.

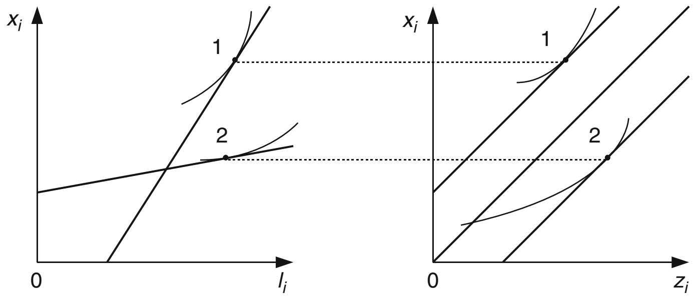

Fig. 7.1 ELIE and envy

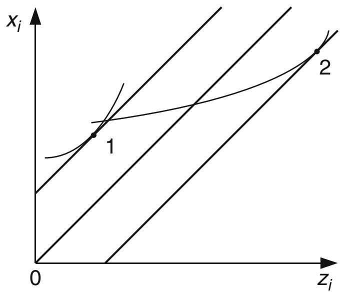

Fig. 7.2 ELIE without envy

ELIE allocation. In the absence of envy at the ELIE allocation, ELIE remains implementable even when individual productivity is private knowledge. If there is at least an individual who envies another one in the population at the ELIE allocation, ELIE is no longer implementable when productivity is private knowledge. The problem is then to find a second-best solution which is "as close as possible" to ELIE. But what is the "closest" solution? At least two different approaches can be considered: the closest in terms of welfare and the closest in terms of transfers. The next two sections examine each of them respectively. Both of them introduce incentive-compatibility constraints in the programme of the policymaker. It is thus worthwhile to examine the implications of these constraints before going further.

By the taxation principle, an income tax schedule corresponds to a mapping

$$\begin{cases} w \longrightarrow \mathbb{R}^+ \times \mathbb{R} \\ w_i \qquad (x_i, z_i) \,, \end{cases} \tag{7.8}$$

which satisfies the incentive-compatibility constraints

$$u(x_i, z_i; w_i) \geq u\left(x_j, z_j; w_i\right), \qquad \forall (i, j) \in \mathscr{I}^2, \tag{7.9}$$

and the tax revenue constraint

$$\sum_{i=1}^{I} z_i \geq \sum_{i=1}^{I} x_i . \tag{7.10}$$

The incentive-compatibility constraints ensure that everyone prefers his own consumption/leisure bundle to that of anyone else. They place structure on feasible income tax schedules. First, the gross income and net income vectors of an incentive-compatible allocation must be non-decreasing in productivity, i.e., such that

$$(x_1, z_1) \leq \cdots \leq (x_I, z_I), \tag{7.11}$$

with $(x_{i-1}, z_{i-1}) << (x_i, z_i)$ if $(x_{i-1}, z_{i-1}) \neq (x_i, z_i)$, $i = 2, \ldots, I$. This condition corresponds to the second-order condition for incentive compatibility in the continuous population framework. Second, given an incentive-feasible consumption vector satisfying (7.11), it proves sufficient to check the downward and upward adjacent incentive compatibility constraints to get an incentive-compatible allocation, providing the Spence–Mirrlees condition holds. This can be formally stated as follows.

Lemma 7.1. *Given $x_1, \ldots, x_I$ and $z_1, \ldots, z_I$ satisfying (7.11),*

$$u(x_i, z_i; w_i) \geq u(x_{i-1}, z_{i-1}; w_i), \qquad i = 2, \ldots, I \tag{7.12}$$

$$u(x_i, z_i; w_i) \geq u(x_{i+1}, z_{i+1}; w_i), \qquad i = 1, \ldots, I-1, \tag{7.13}$$

imply (7.9).

The proof is standard and thus omitted (see Cooper 1984). Many patterns of links between gross-income/consumption bundles satisfy the conditions in Lemma 7.1. This is notably the case if all adjacent *downward* incentive-compatibility constraints are binding, i.e., if (x_i, z_i) and (x_{i+1}, z_{i+1}) are both on person $i+1$'s highest (feasible) indifference curve:

$$u(x_i, z_i; w_{i+1}) = u(x_{i+1}, z_{i+1}; w_{i+1}), \qquad i = 1, \ldots, I-1. \tag{7.14}$$

Allocations satisfying (7.14) are called *simple monotonic chains to the left*. The following result is then easy to establish.

Proposition 7.1. *Let an allocation be a simple monotonic chain to the left satisfying (7.11). Then it satisfies all the incentive compatibility constraints (7.9).*

7.3 The Closest Solution in Terms of Welfare

7.3.1 Methodology

When at least one individual envies another one in the first-best, ELIE is not implementable in the second-best. Designing an incentive-compatible solution then implies a loss in welfare. A first solution is to consider the social weights which generate ELIE in the first-best and then look for the allocation that maximises the corresponding social welfare function subject to the tax revenue constraint (7.10) and the incentive-compatibility constraint (7.9). The optimum allocation is the closest one to ELIE in terms of social welfare in the sense that, given the social weights which generate ELIE in the first-best, it minimises the deadweight loss resulting from asymmetric information. This welfarist approach is certainly clearly distinct from the Kolmian analysis, but it allows us to use the tools and results of the Mirrleesian optimal income tax literature.

Obviously, there is no unicity of the social welfare function which generates ELIE in the first-best information framework. Which function should we pick up? We know that the allocation obtained for the ELIE tax scheme belongs to the frontier of the utility possibility set because it is Pareto-efficient. There is an hyperplane tangent to the Pareto set at the ELIE allocation. The individual social weights $\lambda := (\lambda_1, \ldots, \lambda_I) \in \mathbb{R}^I_{++}$ are defined by this hyperplane and a social welfare function W which is additively separable with respect to these social weights is adopted. Formally, $W : (\mathbb{R}_+ \times [0, w_I])^I \to \mathbb{R}$, with

$$W\left((x_1, z_1), \ldots, (x_I, z_I)\right) = \sum_{i=1}^{I} \lambda_i u\left(x_i, z_i; w_i\right). \tag{7.15}$$

Because $W\left((x_1, z_1), \ldots, (x_I, z_I)\right)$ is homogeneous of degree 1 in λ, the social weights can be normalised without any loss in generality. We choose to set $\mathbb{E}(\lambda) = 1$. Figure 7.3 shows the utility possibility sets in the first-best and the second-best when $I = 2$. The first-best solution is $u^F = \left(u_1^F, u_2^F\right)$. The tangent to the first-best Pareto frontier at u^F has slope $-\lambda_1/\lambda_2$. We consider the family of linear social indifference curves with slope $-\lambda_1/\lambda_2$ and look for the second-best solution. It is obtained at $u^S = \left(u_1^S, u_2^S\right)$ where the second-best Pareto frontier is tangent to the highest (feasible) social indifference curve. Though this approach seems rather natural, it is not free from drawbacks: in particular and as in most of the optimal income tax models, it implies that individual preferences have a cardinal meaning. Yet, it

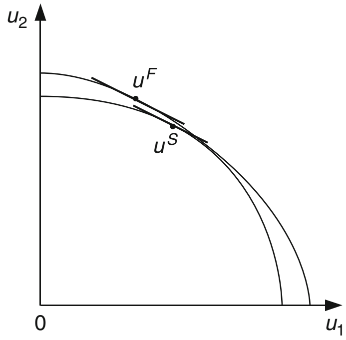

Fig. 7.3 Utility possibility set

is compatible with any finite level of inequality aversion.[2] The problem of the policymaker can thus be stated as follows:

Problem 7.1 (Welfare Approach). *Choose an allocation* $a \in \mathbb{R}_+^I \times [0, w_I]^I$ *which maximises the social welfare function* $W(a)$ *, where the social weights* λ *generate ELIE in the first-best and* $\mathbb{E}(\lambda) = 1$*, subject to the incentive compatibility constraints (7.9) and the tax revenue constraint (7.10).*

Given the utility maximisation programme

$$\max_{(x_i, \ell_i) \in \mathbb{R}_+^2} U(x_i, \ell_i) \text{ s.t. } x_i \leq w_i \ell_i - k(w_i - \mathbb{E}(w)), \tag{7.16}$$

person i's indirect utility is

$$V_i(k) := U\left(w\ell_i^F - k(w_i - \mathbb{E}(w)), \ell_i^F\right). \tag{7.17}$$

Let β_i denote the Lagrange multiplier of person i's budget constraint. The first-order condition with respect to consumption is

$$U_x'(x_i, \ell_i) = \beta_i. \tag{7.18}$$

By the envelope theorem,

$$\beta_i = -\frac{\partial V_i(k)}{\partial[k(w_i - \mathbb{E}(w))]}. \tag{7.19}$$

In the first-best, the social policymaker maximises $W((x_1, z_1), \ldots, (x_I, z_I))$ subject to the tax revenue constraint (7.10). If γ stands for the Lagrange multiplier of this constraint (which must be binding at the social optimum), the first-order condition with respect to consumption is

[2] Note that it is not compatible with the maximin since all social weights are strictly positive.

$$U'_x(x_i, \ell_i) = \frac{\gamma}{\lambda_i}. \tag{7.20}$$

Combining (7.18)–(7.20), one obtains

$$\lambda_i = \gamma \times \left(-\frac{\partial V(w,k)}{\partial T(w,k)} \right)^{-1} \tag{7.21}$$

$$= \frac{\gamma}{U'_x(w_i(\ell_i^F - k) + k\mathbb{E}(w), \ell_i^F)}, \qquad i \in \mathscr{I}. \tag{7.22}$$

In Simula and Trannoy (2011), we have established that, when the marginal utility of consumption is strictly decreasing, these social weights λ are increasing in productivity provided there is ALEP substitutability between consumption and leisure and the substitution effect on labour supply is larger than the income effect.

7.3.2 Features of the Solution

It is possible to derive qualitative features of the solution to Problem 7.1 for different specifications of individual preferences.

We first consider separable preferences, linear with respect to consumption, i.e.,

$$U(x_i, \ell_i) = x_i - v(\ell_i) \text{ with } v' > 0 \text{ and } v'' > 0. \tag{7.23}$$

By (7.22), the social weights which generate ELIE in the first-best are $\lambda_i = \gamma$ for every $i \in \mathscr{I}$. Because preferences are quasilinear in consumption and the social weights λ_i are all equal, the solution to Problem 7.1 is the laissez-faire. Proposition 7.2 casts light on a striking feature of ELIE.

Proposition 7.2. *Consider that individual preferences are quasi-linear in consumption. When ELIE is not envy-free, the closest incentive-compatible allocation in terms of welfare is the laissez-faire.*

We now turn to Cobb–Douglas preferences. Guesnerie and Seade (1982) have examined the solution to the optimal income tax model in a discrete population framework. They consider well-behaved individual preferences, which are assumed to be concave, increasing in consumption and decreasing in leisure. The social objective function is a weighted sum of individual utilities, which is supposed to be defined on and increasing in individual utilities. In this context, designing a nonlinear tax scheme is equivalent for the policymaker to setting a step function in the gross income/consumption space.

A "corner" is thus obtained at every observed gross income/consumption bundle. A nonlinear tax schedule corresponds to a set of corners, each of which being chosen by the individuals who maximise their utility at this bundle. Without loss of generality, these corners can be unambiguously arranged in a South–West/North–East direction. When an additional assumption regarding the shape of the welfare improving transfers is made, the optimal income tax which maximises the objective function subject to the incentive-compatibility constraints and the tax revenue constraint is a simple monotonic chain to the left, provided the Spence–Mirrlees condition is satisfied. In our framework, this assumption may be formulated as follows (see Röell 1985).

Assumption 7.2 (VWR) *For each pair of corners $\left(C_i, C_j\right)$ with $i < j$ (and thus $C_i << C_j$), given $K > 0$ small enough, there exists $\left(\delta^i, \delta^j\right) \in [0, K]^2$ such that it is desirable to distribute $\left(K - \delta^i, -\delta^i\right)$ to agent i and $\left(K - \delta^j, -\delta^j\right)$ away from agent j, provided incentive effects are ignored.*

Proposition 7.3. *Under Assumption VWR, any set of corners $(C_1, \ldots, C_I)$ satisfying (7.11) is welfare-dominated by a simple monotonic chain to the left.*

Proof. See Röell (1985). □

K might be thought of as the quantity of money indirectly transferred from a more to a less productive agent. Under Assumption VWR and the Spence–Mirrlees condition, the optimal income tax schedule must be a simple monotonic chain to the left. We now examine whether the specific shape of the social weights which generate ELIE in the first-best implies Assumption VWR.

For Cobb–Douglas preferences

$$U\left(x_i, \ell_i\right) = \alpha \ln x_i + (1 - \alpha) \ln\left(1 - \ell_i\right), \tag{7.24}$$

in which case

$$\ell_i^F = \max\left\{0, \alpha + (1 - \alpha) \frac{k\left(w_i - \mathbb{E}(w)\right)}{w_i}\right\}. \tag{7.25}$$

By (7.22), when $\ell_i^F > 0$ for every individual,

$$\lambda_i = \gamma\left[w_i(1 - k) + k\mathbb{E}(w)\right]. \tag{7.26}$$

Given our normalisation,

$$\mathbb{E}(\lambda) = \gamma\mathbb{E}(w) = 1 \Leftrightarrow \gamma = \frac{1}{\mathbb{E}(w)}, \tag{7.27}$$

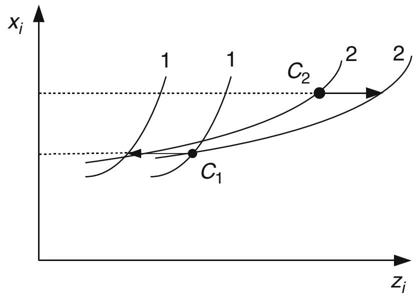

Fig. 7.4 Transfer along the direction given by the gross-income axis

from which

$$\lambda_i = (1-k)\frac{w_i}{\mathbb{E}(w)} + k. \tag{7.28}$$

Let $I = 2$ and consider a transfer $(0, \delta)$ with $\delta > 0$ from person 2 to person 1 (see Fig. 7.4). This transfer – along the direction given by the gross-income axis – satisfies VWR. Let

$$\begin{aligned} W(\delta) = &\left[(1-k)\frac{w_1}{\mathbb{E}(w)} + k\right]\left[\alpha \ln x_1 + (1-\alpha)\ln\left(1 - \frac{z_1 - \delta}{w_1}\right)\right] \\ &+ \left[(1-k)\frac{w_2}{\mathbb{E}(w)} + k\right]\left[\alpha \ln x_2 + (1-\alpha)\ln\left(1 - \frac{z_2 + \delta}{w_2}\right)\right]. \end{aligned} \tag{7.29}$$

Hence,

$$\begin{aligned} W'(\delta) = &\left[(1-k)\frac{w_1}{\mathbb{E}(w)} + k\right]\frac{1-\alpha}{w_1 - z_1 + \delta} \\ &- \left[(1-k)\frac{w_2}{\mathbb{E}(w)} + k\right]\frac{1-\alpha}{w_2 - z_2 - \delta}, \end{aligned} \tag{7.30}$$

from which

$$\begin{aligned} &W'(0) = \\ &(1-\alpha)\left\{\frac{(1-k)\frac{w_1}{\mathbb{E}(w)} + k}{w_1}\frac{1}{1-\ell_1} - \frac{(1-k)\frac{w_2}{\mathbb{E}(w)} + k}{w_2}\frac{1}{1-\ell_2}\right\}. \end{aligned} \tag{7.31}$$

If $W'(0) > 0$, then an increase in δ increases social welfare. This is the case when

$$\begin{aligned} W'(0) > 0 \Leftrightarrow \frac{1-\ell_2}{1-\ell_1} &> \frac{(1-k)\frac{1}{\mathbb{E}(w)} + \frac{k}{w_2}}{(1-k)\frac{1}{\mathbb{E}(w)} + \frac{k}{w_1}} \\ \Leftrightarrow \frac{L_2}{L_1} &> \frac{(1-k)\frac{1}{\mathbb{E}(w)} + \frac{k}{w_2}}{(1-k)\frac{1}{\mathbb{E}(w)} + \frac{k}{w_1}}, \end{aligned} \tag{7.32}$$

where L_i denotes person i's leisure. Equation (7.32) provides us with a sufficient condition for VWR to be satisfied. Using Proposition 7.3, we can therefore proceed as follows to find a solution to Problem 7.1: (a) Assume the solution to Problem 7.1 is a simple monotonic chain to the left. (b) Compute the candidate allocation and check that (7.32) is satisfied. If so, the candidate simple monotonic chain to the left solves Problem 7.1.

7.3.3 An Example

For illustrative purpose, we consider that the population consists of two individuals, who both have the same preferences, represented by the utility function (7.24). The policymaker chooses x_1, z_1, x_2, z_2 to maximise

$$W(x_1, x_2, z_1, z_2) = \lambda_1(k)\left[\alpha \log x_1 + (1-\alpha)\log\left(1 - \frac{z_1}{w_1}\right)\right] + \lambda_2(k)\left[\alpha \log x_2 + (1-\alpha)\log\left(1 - \frac{z_2}{w_2}\right)\right], \quad (7.33)$$

where $\lambda_1(k)$ and $\lambda_2(k)$ are given by (7.26), subject to the incentive compatibility constraints

$$\alpha \log x_1 + (1-\alpha)\log\left(1 - \frac{z_1}{w_1}\right) \geq \alpha \log x_2 + (1-\alpha)\log\left(1 - \frac{z_2}{w_1}\right), \quad (7.34)$$

$$\alpha \log x_2 + (1-\alpha)\log\left(1 - \frac{z_2}{w_2}\right) \geq \alpha \log x_1 + (1-\alpha)\log\left(1 - \frac{z_1}{w_2}\right), \quad (7.35)$$

and the tax revenue constraint

$$z_1 - x_1 + z_2 - x_2 = 0. \quad (7.36)$$

There are three possibilities as regards the incentive compatibility constraints: (a) $\lambda_1 = \lambda_2 = 0$, (b) $\lambda_1 = 0$ and $\lambda_2 > 0$, (c) $\lambda_1 > 0$ and $\lambda_2 = 0$. We apply the strategy described above (for the computational procedure, see the Appendix). For example, assume $w_1 = 1$, $w_2 = 1.5$ and $\alpha = 1/2$. Figure 7.5 shows person 1's and person 2's first-best indirect utilities (denoted U_1 and U_2 respectively) as well as person 2's utility if he were able to choose person 1's leisure/consumption bundle (denoted U_{21}).

For $k \leq 0.166$, person 2 does not envy person 1 and ELIE remains implementable in the second-best. In Simula and Trannoy (2011), we have shown in a continuous population framework that, when gross income is verifiable

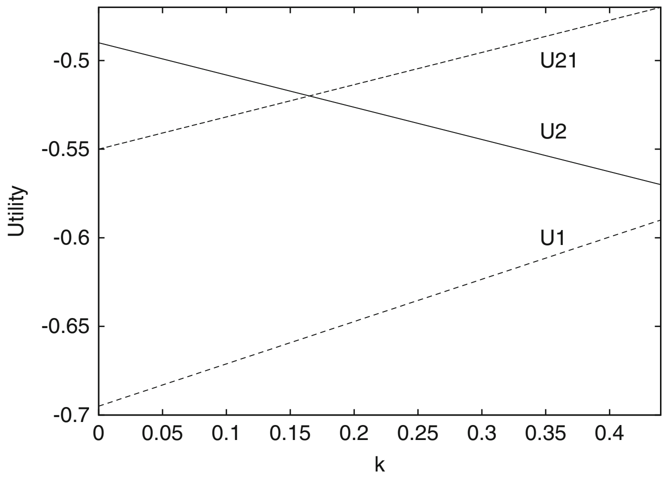

Fig. 7.5 First-best utility levels

and time worked non-verifiable, every individual facing the ELIE tax scheme of degree k has an incentive to understate his true productivity level. Therefore, this tax scheme is not implementable as a direct truthful mechanism in weakly dominant strategies for every value of $k \neq 0$. This shows a basic difference between the continuous and discrete population versions of the optimal income tax model. In the discrete framework and for $k \leq 0.166$ person 2 would have an incentive to mimic the behaviour of an individual with productivity between w_1 and w_2. However, this productivity level does not exist in the population. Hence, this is because he would loose too much in terms of income that person 2 chooses not to mimic person 1.

For $k \geq 0.444$, person 1 chooses to work less than k units of time in the first-best. For $0.166 < k < 0.444$, the second-best optimum allocation is a simple monotonic chain to the left (we can check that (7.32) is verified), i.e., person 2 is indifferent between his consumption/gross-income bundle and that of person 1 while the constraint preventing person 1 from mimicking person 2 is inactive. Figure 7.6 compares the transfers obtained in the first-best (which remain implementable in the second-best for $k \leq 0.166$) with the second-best transfers solution to Problem 7.1. For every $0.166 \leq k \leq 0.444$, the second-best solution is such that person 2 transfers money to person 1. Hence, person 1 is a net recipient of and person 2 a net contributor to the tax policy, as under ELIE. However, the magnitude of the transfers is significantly altered. In the second-best, the transfer to the low-skilled agent is increased compared to the first-best. This is because person 1 now faces a distortive marginal tax rate in order to prevent person 2 from mimicking him (Fig. 7.7).

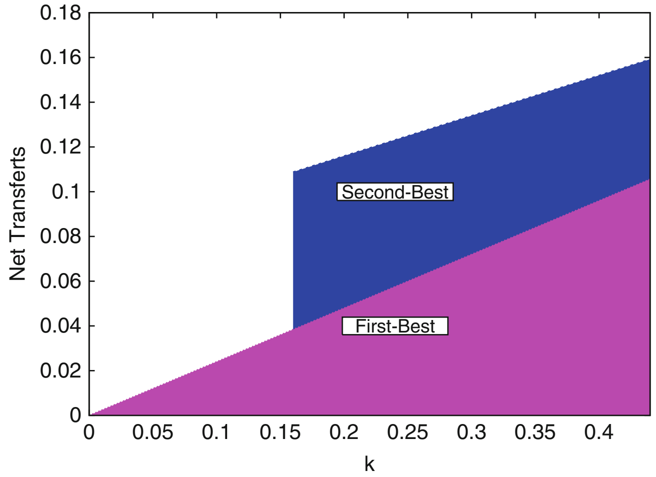

Fig. 7.6 Net transfers to person one

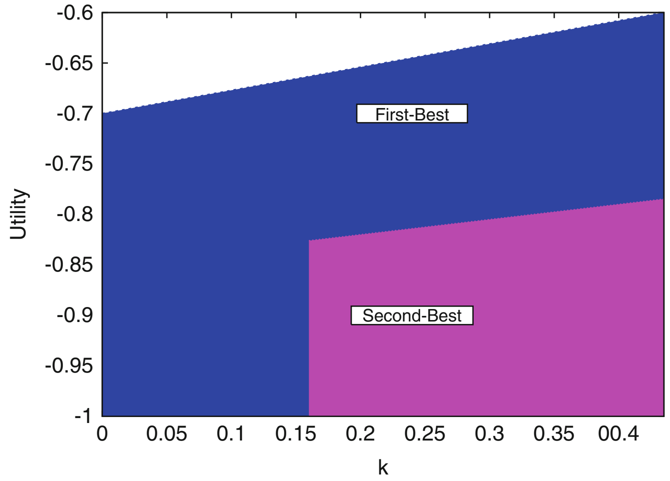

Fig. 7.7 Person one's utility level

7.4 The Closest Solution in Terms of Transfers

As already noted, the welfare approach adopts the viewpoint of the welfarist optimal income tax literature to construct incentive-compatible transfers close to ELIE in the sense specified above. This welfarist perspective seems to significantly differ from Kolm's analytical framework. Moreover, the numerical simulations have shown that the gap between ELIE and the second-best transfers is then rather substantial. In this section, we construct second-best allocations which are as close as possible to ELIE in terms of transfers. Because the focus is on the transfers themselves, we do not require individuals to work at least k units of time. Hence, we concentrate on Kolm's tax schemes of degree k (the ELIE tax schemes of degree k being a particular case of Kolm's tax schemes).

7.4.1 Methodology

The basic idea is to construct a second-best allocation for which all individuals receive the same transfers and pay the same taxes as under ELIE. If this construction is possible, we say that the second-best allocation *replicates* the first-best ELIE transfers. Because we consider cases where at least one individual envies another one in the first-best, the second-best allocation we construct will not coincide with the first-best one; only the taxes and transfers will be the same.

Let us first consider that the population consists of two individuals, i.e., $I = 2$. We are interested in cases where at least one person envies another one since otherwise the first-best allocation is still incentive-compatible in the second-best. According to Fig. 7.1, we thus assume that person 2 envies person 1. To construct a second-best allocation which replicates the ELIE transfers, person 1's and person 2's bundles must stay along their respective first-best budget lines. Given this requirement, how shall we modify the first-best bundles so as to make them incentive-compatible? *A priori*, there are three possibilities: (a) choosing a bundle $(x_2, z_2) >> \left(x_2^F, z_2^F\right)$ for person 2, where $\left(x_i^F, z_i^F\right)$ is person i's first-best bundle, (b) choosing a bundle $(x_1, z_1) << \left(x_1^F, z_1^F\right)$ for person 1 or (c) choosing a combination of (a) and (b). Let us consider person 2's indifference curve through $\left(x_2^F, z_2^F\right)$. Because $s\left(x_2^F, z_2^F, w_2\right) = 1$, it is impossible to find higher indifference curves which intersect person 2's first-best budget line. The only possibility is to find lower indifference curves. But that does not induce incentive-compatibility. In fact, the only possibility is thus to decrease person 1's utility. So, person 2 gets his first-best utility level $\left(x_2^F, z_2^F\right)$. The minimum decrease in person 1's utility is obtained when $\left(x_1^F, z_1^F\right)$ is at the junction of person 2's indifference curve through $\left(x_2^F, z_2^F\right)$ and person 1's budget line. Hence, the solution must be (b). The implication is that person 2 receives his first-best bundle while person 1's choices are distorted (to the minimum). This is illustrated on the left-hand side of Fig. 7.8. This construction is not always possible as shown by the right-hand side. Indeed, there might be not intersection between person 2's indifference curve through his first-best bundle and person 1's first-best

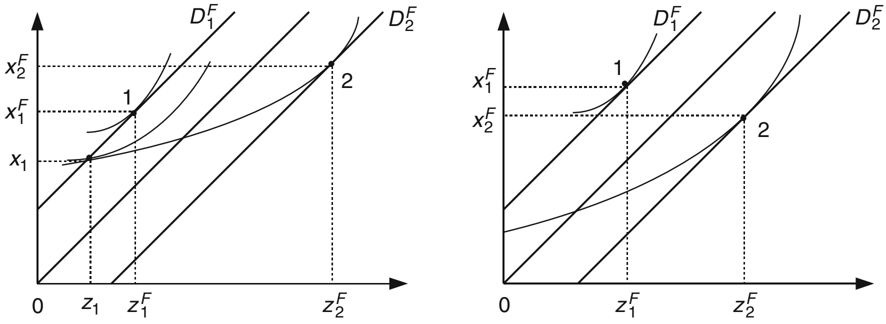

Fig. 7.8 Closest solution in transfers: methodology

budget line. In that case, ELIE transfers cannot be replicated using a second-best income tax scheme.

Now, assume $I = 3$. Person 3 receives his first-best bundle. The minimum incentive-compatible decrease in person 2's utility is obtained when he receives the bundle at the junction of his budget line and person 3's highest (feasible) indifference curve. Now, let us consider person 2 and person 1 and keep person 2's bundle fixed. Using the same argument as in the two-person case, it is clear that incentive-compatibility now requires a decrease in person 1's utility. This reduction is minimised when person 1's bundle is at the junction of person 2's indifference curve through his second-best bundle and person 1's first-best budget line. By induction, the same procedure applies recursively when $I > 3$, the loss in utility being minimised at each step. Recall an allocation for which person i is indifferent between his own bundle and that designed for the closest less productive person, i.e., for which

$$u(x_i, z_i; w_i) = u(x_{i-1}, z_{i-1}; w_i), \qquad i = 2, \ldots, I, \tag{7.37}$$

is called a simple monotonic chain to the left after Guesnerie and Seade (1982). The problem addressed in this section can thus be summarised as follows.

Problem 7.2. Construct a simple monotonic chain to the left for which

$$z_i - x_i = k(w_i - \mathbb{E}(w)), \qquad i = 1, \ldots, I. \tag{7.38}$$

In fact, because the Spence–Mirrlees condition is met, the satisfaction of all local incentive-compatibility constraints induces the satisfaction of all global incentive-compatibility constraints (Cooper 1984). Hence, pairwise comparisons of adjacent bundles are sufficient to get incentive-compatibility.

In the above construction, the second-best bundles must also be on the first-best budget lines. By construction, every individual $i < I$ incurs the minimum loss in utility so as to replicate the ELIE transfers by the means of an income tax. Consequently:

Proposition 7.4. *If there exists a solution to Problem 7.2, then it is unique and Pareto-efficient in the constraint set.*

7.4.2 The Geometric Construction

To construct a solution to Problem 7.2, it is useful to define

$$\mathscr{D}_i(k) := \{(x,z) \in \mathbb{R}_+ \times [0, w_I] : x = z - k(w_i - \mathbb{E}(w))\}, \quad (7.39)$$

as person i's budget line when ELIE transfers of degree k are implemented in a first-best environment. Since there is no distortion at the top, the most productive individual must receive his first-best bundle $\left(x_I^F, z_I^F\right)$ solution to

$$\max u(x,z;w_I) \text{ s.t. } (x,z) \in \mathscr{D}_I(k). \quad (7.40)$$

So, $(x_I, z_I) = \left(x_I^F, z_I^F\right)$. It is then possible to determine person $I-1$'s bundle (x_{I-1}, z_{I-1}). On the one hand, person I must be indifferent between (x_I, z_I) and (x_{I-1}, z_{I-1}) because we are looking for a simple monotonic chain to the left. Therefore, (x_{I-1}, z_{I-1}) must belong to the set

$$\mathscr{U}(x_I, z_I; w_I) := \{(x,z) \in [0, x_I] \times [0, z_I] : u(x,z;w_I) = u(x_I, z_I; w_i)\}. \quad (7.41)$$

On the other hand, the constraint $z_{I-1} - x_{I-1} = k(w_{I-1} - \mathbb{E}(w))$ must be satisfied, which means that the bundle (x_{I-1}, z_{I-1}) must belong to person $I-1$'s first-best budget line $\mathscr{D}_{I-1}(k)$. As a consequence,

$$(x_{I-1}, z_{I-1}) = \mathscr{U}(x_I, z_I; w_I) \cap \mathscr{D}_{I-1}(k). \quad (7.42)$$

The construction then proceeds recursively until (x_1, z_1) is obtained. Figure 7.9 illustrates it for a two-person population (in this figure, k is suppressed from the notation; x^F, z^F and T^F correspond to first-best levels; x, z and T to second-best levels; indifference curves are labeled by productivity levels). The two budget lines for the low-skill individual cross at the new equilibrium for this individual.[3]

[3] A difficulty that might arise is that the intersection may be obtained for negative values of z_i. In that case, individual preferences just need to be reinterpreted: a negative quantity of labour would then correspond to a transfer in labour to person i.

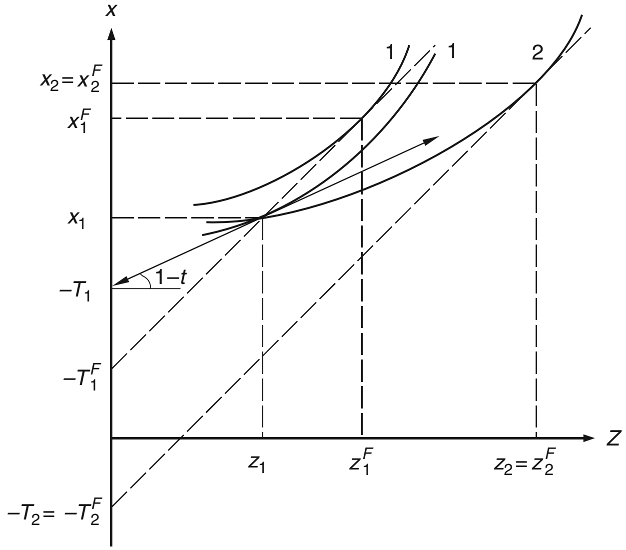

Fig. 7.9 Closest solution in transfers: geometric construction

7.4.3 An Example

Complete solutions for the approach in terms of transfers are now provided. To this aim, it is assumed that the population consists of two individuals ($I = 2$) who both have Cobb–Douglas preferences,

$$U(x, \ell) = \alpha \ln x + (1-\alpha)\ln(1-\ell), \ \alpha \in (0,1). \tag{7.43}$$

Person 2's choices are not distorted. He thus gets his first-best bundle

$$\left(x_2^F, z_2^F\right) = \left(\alpha\left(w_2(1-k) + k\mathbb{E}(w)\right), \alpha\theta_2 + (1-\alpha)k\left(w_2 - \mathbb{E}(w)\right)\right) \tag{7.44}$$

So, his indirect utility amounts to

$$u\left(x_2^F, z_2^F; w_2\right) := \ln\left[\alpha^\alpha(1-\alpha)^{1-\alpha}\left(w_2(1-k) + k\mathbb{E}(w)\right)^\alpha\left(1-k+k\frac{\mathbb{E}(w)}{w_2}\right)^{1-\alpha}\right] = \bar{u}_2. \tag{7.45}$$

Person 1's second-best bundle must belong to the left-hand part of person 2's highest indifference curve,

$$\mathscr{U}\left(x_2^F, z_2^F; w_2\right) := \left\{(x,z) \in \left[0, x^F(w_2)\right] \times \left[0, z^F(w_2)\right] : x = \exp\left(\frac{\overline{u}_2}{\alpha}\right)\left(1 - \frac{z}{w_2}\right)^{1-\frac{1}{\alpha}}\right\}. \tag{7.46}$$

As a consequence, it is the unique solution in (x, z) in $\left[0, x^f(w_2)\right] \times \left[0, z^f(w_2)\right]$ (if any) to the following system:

$$\begin{cases} z = \alpha(1-\alpha)^{\frac{1}{\alpha}-1} w_2^{1-\frac{1}{\alpha}} \left(w_2 + k\frac{w_1 - w_2}{2}\right)^{\frac{1}{\alpha}} \left(1 - \frac{z}{w_2}\right)^{1-\frac{1}{\alpha}} + k\frac{w_1 - w_2}{2}, \\ x = z - t(w_1; w, k). \end{cases} \tag{7.47}$$

For illustrative purposes, we consider the same economy as in the closest-solution-in-welfare approach: $\alpha = 1/2$, $w_1 = 1$ and $w_2 = 3/2$. In this case, the first equation in (7.47) is

$$z = \frac{1}{8}\frac{(3 - k/2)^2}{3 - 2z} - \frac{k}{4}. \tag{7.48}$$

Hence, the income tax scheme is characterised by

$$\begin{cases} (x_1, z_1) = \left(\frac{k}{4} + \frac{1}{8}\left(6 - 2\sqrt{6k} - k\right), \frac{1}{8}\left(6 - 2\sqrt{6k} - k\right)\right), \\ (x_2, z_2) = \left(\frac{3}{4} - \frac{k}{8}, \frac{3}{4} + \frac{k}{8}\right). \end{cases} \tag{7.49}$$

It is therefore possible to make the ELIE transfers incentive-compatible for every $k \in (0, 1)$. Gross income and consumption levels are illustrated in Fig. 7.10, panels (1) and (2); person 1's marginal tax rates and second-best lump-sum transfers on panels (3) and (4). The implicit marginal tax rate faced by person 1 is given by

$$T'(x_1, z_1; w_1) = 1 - \frac{1-\alpha}{\alpha}\frac{x_1}{w - z_1} = \frac{4\left(\sqrt{6k} - 1\right)}{2 + 2\sqrt{6k} + k}. \tag{7.50}$$

Hence, the marginal tax rate faced by person 1 increases with k. Its sign is given by

$$T'(x_1, z_1; w_1) \geq 0 \Leftrightarrow k \geq 1/6 \simeq 0.1666, \tag{7.51}$$

$$T'(x_1, z_1; w_1)' = 0 \text{ for } k \leq 1/6. \tag{7.52}$$

For $k \leq 1/6$, ELIE is indeed incentive-compatible as previously noted; hence both agents face a zero marginal tax rate.

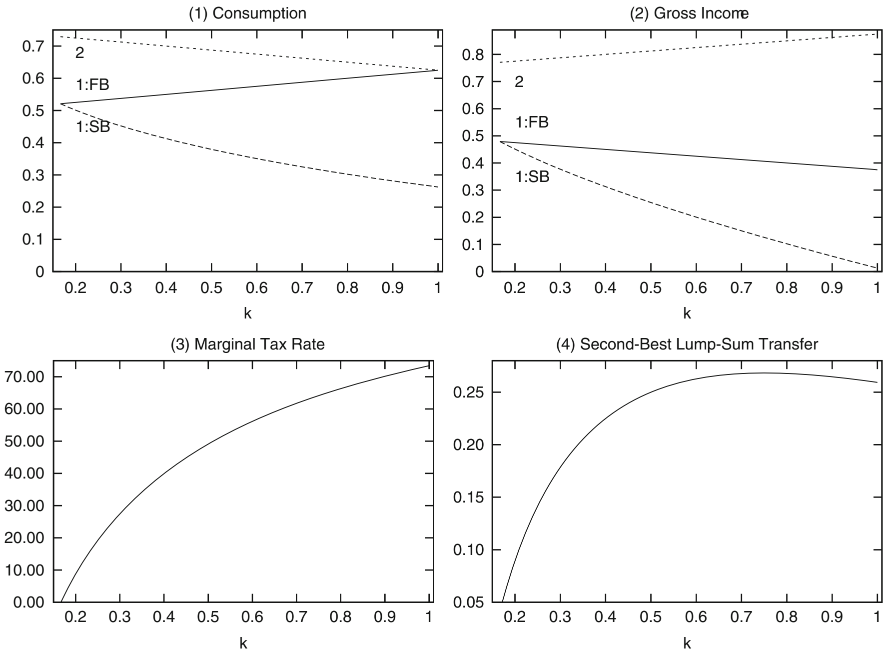

Fig. 7.10 Closest solution in transfers: example economy

7.5 Concluding Comments

When they exist, the closest solution in terms of welfare and the closed solution in terms of transfers are both Pareto-efficient. Yet, they do not belong to the same Pareto set. Indeed, the constraint set of Problem 7.2 is included in that of Problem 7.1.

The income tax corresponding to the closest solution in terms of welfare may be substantially different from ELIE. This is notably the case for quasilinear preferences. For every value of k for which there is envy in the first-best, the closest solution is the laissez-faire, which is only obtained for $k = 0$ in the first-best. However, Kolm analysis puts the stress on the very specific shape of the ELIE transfers and does not adopt a welfarist perspective. This justifies the second approach we have proposed, in which the second-best transfers exactly correspond to the first-best ones. Our procedure yields a unique Pareto-efficient outcome in the constraint set.

Appendix

The Lagrangian of this optimisation problem is

$$\begin{aligned} L = {} & \lambda_1 \left[\alpha \log x_1 + (1-\alpha) \log \left(1 - \frac{z_1}{w_1} \right) \right] \\ & + \lambda_2 \left[\alpha \log x_2 + (1-\alpha) \log \left(1 - \frac{z_2}{w_2} \right) \right] \\ & + \mu_1 \left[\alpha \log x_1 + (1-\alpha) \log \left(1 - \frac{z_1}{w_1} \right) - \alpha \log x_2 - (1-\alpha) \log \left(1 - \frac{z_2}{w_1} \right) \right] \\ & + \mu_2 \left[\alpha \log x_2 + (1-\alpha) \log \left(1 - \frac{z_2}{w_2} \right) - \alpha \log x_1 - (1-\alpha) \log \left(1 - \frac{z_1}{w_2} \right) \right] \\ & + \gamma \left[z_1 - x_1 + z_2 - x_2 \right]. \end{aligned} \tag{A.1}$$

The first-order conditions for a maximum are

$$\frac{\partial L}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{\alpha}{\gamma} \left(\lambda_1 + \mu_1 - \mu_2 \right), \tag{A.2}$$

$$\frac{\partial L}{\partial z_1} = 0 \Leftrightarrow \frac{\lambda_1 + \mu_1}{z_1 - w_1} - \frac{\mu_2}{z_1 - w_2} = -\frac{\gamma}{1-\alpha}, \tag{A.3}$$

$$\frac{\partial L}{\partial x_2} = 0 \Leftrightarrow x_2 = \frac{\alpha}{\gamma} \left(\lambda_2 + \mu_2 - \mu_1 \right), \tag{A.4}$$

$$\frac{\partial L}{\partial z_2} = 0 \Leftrightarrow \frac{\lambda_2 + \mu_2}{z_2 - w_2} - \frac{\mu_1}{z_2 - w_1} = -\frac{\gamma}{1-\alpha}, \tag{A.5}$$

$$\mu_1 \geq 0 \ \ (= 0 \text{ if (7.34) is not binding}), \tag{A.6}$$

$$\mu_2 \geq 0 \ \ (= 0 \text{ if (7.35) is not binding}). \tag{A.7}$$

Substituting (A.2) and (A.4) in the tax revenue constraint (7.36),

$$z_1 + z_2 = x_1 + x_2 = \frac{\alpha}{\gamma} \left(\lambda_1 + \lambda_2 \right) = \frac{\alpha}{\gamma} \Leftrightarrow z_1 = \frac{2\alpha}{\gamma} - z_2, \tag{A.8}$$

because $\lambda_1 + \lambda_2 = 2$. Moreover, note that

$$x_1 + x_2 = \frac{2\alpha}{\gamma} \Leftrightarrow x_1 = \frac{2\alpha}{\gamma} - x_2. \tag{A.9}$$

Case (i). We first examine whether the first-best solution remains incentive-compatible when individual productivity becomes private knowledge. In the first-best, person i's labour and consumption choices are respectively given by

$$\ell_1^F = \begin{cases} \alpha + (1-\alpha)k\left(1 - \frac{\mathbb{E}(w)}{w_1}\right) & \text{if } w_1 > w_0, \\ 0 & \text{if } w_1 \leq w_0, \end{cases} \tag{A.10}$$

$$x_1^F = \begin{cases} \alpha[w_1(1-k) + k\mathbb{E}(w)] & \text{if } w_1 > w_0, \\ k\left[\mathbb{E}(w) - w_1\right] & \text{if } w_1 \leq w_0, \end{cases} \tag{A.11}$$

$$\ell_2^F = \alpha + (1-\alpha)k\left(1 - \frac{\mathbb{E}(w)}{w_2}\right) \tag{A.12}$$

$$x_2^F = \alpha\left[w_2(1-k) + k\mathbb{E}(w)\right], \tag{A.13}$$

where

$$w_0 := \frac{k}{k + \frac{\alpha}{1-\alpha}}\mathbb{E}[w], \tag{A.14}$$

is a productivity threshold under which individuals are idle, provided that $w_0 \geq \underline{\theta}$. We check whether there is at least one individual which envies another one.

Case (ii). The incentive-compatibility constraint preventing the highly skilled from mimicking the low skilled is binding. By (A.5),

$$z_2 = w_2 - \frac{1-\alpha}{\gamma}(\lambda_2 + \mu_2). \tag{A.15}$$

Moreover, equalising (A.3) and (A.5),

$$\frac{\lambda_1}{z_1 - w_1} - \frac{\mu_2}{z_1 - w_2} = \frac{\lambda_2 + \mu_2}{z_2 - w_2}, \tag{A.16}$$

in which (A.8) is substituted to get rid of z_1:

$$\frac{\lambda_1}{\frac{\alpha}{\gamma} - z_2 - w_1} - \frac{\mu_2}{\frac{\alpha}{\gamma} - z_2 - w_2} = \frac{\lambda_2 + \mu_2}{z_2 - w_2}. \tag{A.17}$$

Now, use (A.15) to get rid of z_2,

$$\frac{\mu_2}{\frac{\alpha}{\gamma} + \frac{1-\alpha}{\gamma}(\lambda_2 + \mu_2) - 2\theta_2} - \frac{\lambda_1}{\frac{\alpha}{\gamma} + \frac{1-\alpha}{\gamma}(\lambda_2 + \mu_2) - (w_1 + w_2)} = \frac{\lambda_2 + \mu_2}{\frac{1-\alpha}{\gamma}(\lambda_2 + \mu_2)}, \tag{A.18}$$

which is an equation in μ_2. Solving (A.2), one gets μ_2 as an implicit function of γ, given α, w_1, w_2 and k,

$$\mu_2 = \mu_2(\gamma). \tag{A.19}$$

Plugging $\mu_2(\gamma)$ in (A.15), z_2 is obtained as a function of γ, $z_2 = z_2(\gamma)$, from which $z_1 = z_1(\gamma)$ because of (A.8).

Substitution of $z_1(\gamma)$ and $z_2(\gamma)$ in the binding incentive-compatibility constraint (7.35) yields:

$$\alpha \log x_2 + (1-\alpha)\log\left(1 - \frac{w_2 - \frac{1-\alpha}{\gamma}(\lambda_2 + \mu_2\gamma))}{w_2}\right)$$
$$= \alpha \log\left(\frac{\alpha}{\gamma} - x_2\right) +$$
$$(1-\alpha)\log\left(1 - \frac{\frac{\alpha}{\gamma} + \frac{1-\alpha}{\gamma}(\lambda_2 + \mu_2(\gamma)) - w_2}{w_2}\right). \quad \text{(A.20)}$$

Solving it, x_2 is obtained as a function of γ, $x_2 = x_2(w)$. Using Then, x_1 is determined as $x_1(\gamma) = \frac{\alpha}{\gamma} - x_2$. Finally, we choose the value of γ which maximises $W(x_1(\gamma), x_2(\gamma), z_1(\gamma), z_2(\gamma))$ and check that (7.34) is satisfied. We check whether condition (7.32) is satisfied. If not, we would also consider the last case and then compare (i)–(iii).

Case (iii). The incentive compatibility constraint preventing the low type from mimicking the high type is active. The procedure follows the same lines as in case (i), but (7.34) now plays the part of (7.35).

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