ENS de Lyon
General Equilibrium: Exercises

Exercise 1

Consider a two-individual, two-good economy. The preferences of individual A and individual B over good 1 and good 2 are represented by the utility function:

$$u(x_1, x_2) = x_1 x_2,\ x_1 \geq 0, x_2 \geq 0,$$

where x_i denotes the quantity of good i consumed by the individual. There are 3 units of each good in the economy.

1. Characterize the set of Pareto optimal allocations.

The 3 units of each good are initially shared as follows between the two individuals: A has one unit of good 1 and 2 units of good 2; B has 2 units of good 1 and 1 unit of good 2.

2. Determine the Walrasian equilibria.
3. Represent the economy in the Edgeworth box. Comment.

Exercise 2

Consider a two-person, two-good pure exchange economy. A's preferences over consumption bundles (x_1, x_2) are represented by the utility function

$$U^A(x_1, x_2) = x_1 x_2$$

where $x_1 \geq 0$ denotes the quantity of good 1 and $x_2 \geq 0$ the quantity of good 2. B's preferences over consumption bundles are represented by the utility function

$$U^B(x_1, x_2) = x_1 + x_2.$$

The initial endowments in goods 1 and 2 are respectively $\Omega^1 = 1$ and $\Omega^2 = 2$.

1. Represent this economy in the Edgeworth box.
2. Define a Pareto optimum allocation for this economy.
3. Derive the set of Pareto optimum allocations. Represent it on the graph depicted in Question 1.
4. Would you have enough information to compute the general equilibria of this economy? Explain the difference between a Pareto optimal allocation and a general equilibrium.

Exercise 3

Consider a two-person two-good pure exchange economy. Person A's and person B's preferences over consumption bundles $(x_1, x_2) \in \mathbb{R}_+^2$ are represented by the following utility functions:

$$\begin{aligned} U^A(x_1, x_2) &= x_1 + \ln x_2, \\ U^B(x_1, x_2) &= x_1 + 2\ln x_2. \end{aligned}$$

There are initially 5 units of good 1 and 3 units of good 2 in the economy.

1. Determine the set of Pareto optimum allocations. Represent it in the Edgeworth box.
2. Consider any Pareto optimum allocation $\left(x_1^A, x_2^A, x_1^B, x_2^B\right) >> 0$, where x_i^j stands for the quantity of good i available to person j.
   (a) Establish that it is a general equilibrium associated with a price vector and a distribution of person A's and person B's exogenous incomes, denoted ω^A and ω^B respectively. Without loss of generality, the price of good 1 will be normalized to 1.
   (b) Determine the Pareto optimum allocation for which the distribution of incomes is egalitarian in the sense that $w^A = w^B$.
   (c) Person A has 4 units of good 1 and 2 units of good 2. Person B has 1 unit of each good. What are the income transfers from A to B which lead to the egalitarian equilibrium?

Exercise 4

Consider a two-good (1 and 2), two-individual (A and B) economy. Let $x = (x_1, x_2) \in \mathbb{R}_+ \times \mathbb{R}_{++}$ be a consumption bundle. A's preferences are represented by the utility function defined over consumption bundles, $u_A : \mathbb{R}_+ \times \mathbb{R}_{++} \longrightarrow \mathbb{R}$ with

$$u_A(x) = x_1 + \ln x_2,$$

B's preferences are represented by $u_B : \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}$ with

$$u_B(x) = \min\{x_1, x_2\}.$$

A and B's endowments in goods 1 and 2 are $\omega^A = (3, 0)$ and $\omega^B = (0, 3)$.

1. Compute the Walrasian equilibrium.
2. A productive sector is now added to the pure exchange economy considered above. The productive sector consists of one firm which transforms a quantity y_1 of good 1 in a quantity y_2 of good 2 according to the technology
$$y_2 = f(y_1) = a y_1, \text{ where } a > 0.$$
   (a) Define a Walrasian equilibrium for this economy.
   (b) Compute a Walrasian equilibrium E^2 for this economy. Comments?

Exercise 5

Let us consider a two-good, one-consumer (A) economy. x is the sole consumption good and is leisure L (in hours). A's preference relation is represented by the utility function $U(x,L) = x^{\alpha}L^{1-\alpha}$, with $0 < \alpha < 1$. His initial endowment is $(0,\overline{L})$. So, labour is defined as $\ell := \overline{L} - L$. A firm uses labour ℓ to produce the consumption good x.

1. Examine the Walrasian equilibria of this economy when the technology is $x = \ell^{1/2}$.
2. Examine the Walrasian equilibria of this economy when the technology is $x = a\ell$, $a > 0$.
3. The technology is $x = \ell^2$. Represent the production possibility set and draw some indifference curves in the (ℓ, x)-space. Determine the Pareto optimal allocation of this economy. Represent it in the same graph. Show that the Pareto optimal allocation cannot be decentralized as a Walrasian equilibrium. Comment.