The objectives of the producer

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Outline

Minimizing Costs

- A firm does not always aim at maximizing profits. Yet, **whatever its final objective**, a rational producer must minimize the production costs.
- **Given the chosen output level, what is the input combination for which the production cost is minimum?**
- The **time** dimension is important when answering this question.

Variable and Constrained Inputs

Two kinds of inputs:

- **variables inputs**: the amount used by the firm can be varied at will by the firm.
- **constrained inputs**: at least period of time to vary the amount of constrained inputs used by the firm.

Short Run and Long Run

- **In the short run**, the firm usually employs a combination of variable and constrained inputs.
  - Example of labour and capital.
  - Implication: the firm can only fully decide about the amounts of variable inputs it uses whilst the amounts of constrained inputs are fixed or subject to availability constraints.
- **In the long run**, all inputs are variables.

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Long-Run Cost Minimization

- A firm
- Two **variable** inputs, labelled 1 and 2. Prices: p_1 and p_2. Exogenously given. Used in quantities z_1 and z_2.
- Objective: producing q units of output and minimize production cost.

Problem (Long-Run Cost Minimization)

Choose the input combination (z_1, z_2), *to produce output* q, *which minimizes the production cost* $p_1 z_1 + p_2 z_2$.

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Graphical Solution

- Production function $f(z_1, z_2)$. Isoquant of level q: $IS_q := \left\{(z_1, z_2) \in \mathbb{R}_+^2 : f(z_1, z_2) = q\right\}$. f twice continuously differentiable. Decreasing marginal products for input 1 and input 2.
- Isocost curve of level c:

$$C := \left\{(z_1, z_2) \in \mathbb{R}_+^2 : p_1 z_1 + p_2 z_2 = c\right\}.$$

  - Here, isocost curves are isocost lines. Equation in (z_1, z_2)-space: $z_2 = \frac{C}{p_2} - \frac{p_1}{p_2} z_1$.
  - Infinity of isocosts lines, with cost c ranging from 0 to infinity.

ftbpFU4.2903in2.693in0ptLong-run cost minimization. The cost-minimizing input combination is obtained at the tangency point between the isoquant of level q (q is exogenously given) and the lowest indifference curve (here C_1).Figure1Figure

Cost-Minimizing Input Combination

Theorem

Costs are minimized at the input combination (z_1^*, z_2^*) *which satisfies*

$$MRTS_{12}(z_1^*, z_2^*) = \frac{p_1}{p_2}. \quad (1)$$

- (z_1^*, z_2^*): functions of p_1, p_2 and q.
- We denote them by $z_1^*(p_1, p_2, q)$ and $z_2^*(p_1, p_2, q)$.
- Called **conditional input demands**.

Alternative Characterization

- By definition

$$MRTS_{12}(z_1, z_2) = \frac{\partial f(z_1, z_2) / \partial z_1}{\partial f(z_1, z_2) / \partial z_2}.$$

- Hence, at the optimum,

$$\frac{\partial f(z_1^*, z_2^*) / \partial z_1}{\partial f(z_1^*, z_2^*) / \partial z_2} = \frac{p_1}{p_2} \Longleftrightarrow \frac{\partial f(z_1^*, z_2^*) / \partial z_1}{p_1} = \frac{\partial f(z_1^*, z_2^*) / \partial z_2}{p_2}.$$

- **Interpretation.** In the cost-minimizing input combination, the "marginal products in value" $\frac{\partial f(z_1,z_2)/\partial z_1}{p_1}$ and $\frac{\partial f(z_1,z_2)/\partial z_2}{p_2}$ are equal. Why?

Outline

Lagrange Method

Body Math

Problem

$\min_{z_1 \geq 0, z_2 \geq 0} p_1 z_1 + p_2 z_2$ *such that.* $f(z_1, z_2) = q$.

- Lagrangian:

$$L = p_1 z_1 + p_2 z_2 + \lambda \left[f(z_1, z_2) - q \right].$$

- Necessary conditions:

$$\frac{\partial L}{\partial z_1} = p_1 + \lambda \frac{\partial f(z_1, z_2)}{\partial z_1} = 0, \quad (2)$$

$$\frac{\partial L}{\partial z_2} = p_2 + \lambda \frac{\partial f(z_1, z_2)}{\partial z_2} = 0. \quad (3)$$

- Using (2) and (3):

$$\frac{\partial f(z_1, z_2) / \partial z_1}{p_1} = \frac{\partial f(z_1, z_2) / \partial z_2}{p_2}.$$

Outline

1. MINIMIZING COSTS
   - Long-Run Cost Minimization
     - The Firm's Decision Problem
     - Graphical Solution
     - Lagrange Method
     - Comparative Statics of the Cost-Minimizing Solution
     - Cost Functions
   - Short-run cost minimization
     - Constraint on input 2 and cost minimization
     - Solution
2. MAXIMIZING PROFITS
   - Long-run Profit Maximization
     - The Firm's Decision Problem
     - The Firm's Optimum Production Plan
     - Long-Run Supply Function
   - Short-Run Profit Maximization
3. CONCLUSION: THE MARKET SUPPLY

Changing input prices

- If **both inputs' prices change in the same proportion**. No effect on the choice of the cost-minimizing input combination because the slope of the isocost lines, $-p_1/p_2$, is not modified.
- **Impact of increasing the price of an input (or of both inputs but in different proportions)**, all other things being equal. Illustration with increase in p_1 and decrease in p_2.

ftbpFU4.3336in2.7069in0ptImpact of a Change in Input PricesFigure3Figure

Changing output level

ftbpFU3.403in2.4768in0ptImpact of a Variation in the Output Level q on the Cost-Minimizing Input Combinations.Figure2Figure

Outline

Cost Function

Definition (Long-Run Cost Function)

The long-run cost function of the firm is defined has

$$C(p_1, p_2, q) = p_1 z_1^* + p_2 z_2^*.$$

Under the condition that all inputs are variables. It is clear that:

$$\begin{aligned}
\frac{\partial C(p_1, p_2, q)}{\partial p_1} &> 0, \\
\frac{\partial C(p_1, p_2, q)}{\partial p_2} &> 0, \\
\frac{\partial C(p_1, p_2, q)}{\partial q} &> 0.
\end{aligned}$$

ftbpFU3.5206in3.4878in0ptLong-run Cost Functions: Total Cost, Average Cost, Marginal CostFigure4Figure

Outline

Short-Run Cost Minimization

- **Two inputs**, 1 and 2, with prices p_1 and p_2, used in quantities z_1 and z_2, to produce q units of output (q is given).
- Input 1 is a **variable** input.
- Input 2 is a **constrained input**.
- The constraint on input 2 may take a variety of forms.

Outline

Constrained Input: Indivisible Input

The firm may be **constrained to use exactly a predetermined amount of input** z_2, denoted $\overline{z}_2$.

Problem

Choose the input combination (z_1, z_2) *to produce output* q *which minimizes the production cost* $p_1 z_1 + p_2 z_2$ *subject to the constraint* $z_2 = \overline{z}_2$.

- the only control variable is z_1.
- Impossible to use less of input 2 than $\overline{z}_2$. $\Rightarrow$ input 2 is said to be "indivisible".

Constrained Inputs: Fixed Ceiling

When the constrained inputs are divisible, it is always possible for the firm to use less of them. Let $\overline{z}_2$ be a **fixed ceiling** on the amount of input 2 currently available.

Problem

Choose the input combination (z_1, z_2) *to produce output* q *which minimizes the production cost* $p_1 z_1 + p_2 z_2$ *subject to the constraint* $z_2 \leq \overline{z}_2$.

- In this minimization problem the production cost is equal to $p_1 z_1 + p_2 z_2$.
- **If there is a ceiling on the units of input** 2 **available to the firm, the firm only pays the units of inputs** 1 **and** 2 **it actually uses.**

Constrained Inputs: Fixed Cost

Assume **input** 2 **is divisible** and the firm

- has already contracted to pay $p_2\bar{z}_2$ to use input 2;
- already owns $\bar{z}_2$ units of input 2 but caanot sell $\bar{z}_2 - z_2$ in the short run if it wants to use $z_2 < \bar{z}_2$.

In both cases, **the production cost is** $p_1 z_1 + p_2\bar{z}_2$.

Problem

Choose the input combination (z_1, z_2) *to produce output* q *which minimizes the production cost* $p_1 z_1 + p_2\bar{z}_2$ *subject to the constraint* $z_2 \leq \bar{z}_2$.

- The existence of a fixed input gives rise to a fixed cost because **the firm must pay** $p_2\bar{z}_2$ **for input** 2 **even if it uses** $z_2 \leq \bar{z}_2$ **in the production process**.
- The fixed cost is equal to $p_2\bar{z}_2$.

Outline

ftbpFU2.4863in3.0208in0ptCeilingFigure7Figure ftbpFU4.0776in2.4708in0ptFixed CostFigure8Figure

Outline

The Final Objective of the Firm

- Up to now, we did not discuss the objective of the firm.
- We now address the issue of the determination of the output level.
- Most standard objective considered in the economic literature: the producer aims at maximizing its profits.

Outline

Long-Run Profit Maximization

- Firm's decision problem: plan an output and input combination to maximize profits.
- Long-run: all inputs are variable.
- For convenience: two inputs, 1 and 2, with prices p_1 and p_2. Ouput produced in quantity y and sold at price p (p exogenously given; idea: "small" firm).

Outline

Two Formulations of the Firm's Decision Problem

Problem (Profits Maximization)

Chose an output level $y \in \mathbb{R}_+$ *to maximize the firm's profits*

$$\pi := py - C(p_1, p_2, y).$$

Problem (Profits Maximization)

Chose a production plan $(y, z_1, z_2) \in \mathbb{R}^3_+$ *to maximize the firm's profits*

$$\pi := py - \sum_{i=1}^{2} p_i z_i$$

subject to the technology constraint

$$y \leq f(z_1, z_2).$$

Outline

Firm's Optimum Using the First Formulation

- Necessary condition for an interior maximum:

$$\frac{\partial \pi}{\partial y} = p - \frac{\partial C\left(p_1, p_2, y\right)}{\partial y} = 0 \Leftrightarrow p = \frac{\partial C\left(p_1, p_2, y\right)}{\partial y}. \qquad (4)$$

Theorem

When the output level is chosen optimally, the cost of producing an extra unit of output is equal to its price.

- **This condition may not be sufficient.**
  - $y > 0$ satisfying (4) may be a minimum or an inflection point.
  - To ensure that y is a (local) maximum, we must check that the firm's profit function is locally concave. This is the case when

$$\frac{d^2 \pi}{dy^2} = -\frac{\partial C^2\left(p_1, p_2, y\right)}{\partial y^2} < 0 \Leftrightarrow \frac{\partial C'\left(p_1, p_2, y\right)}{\partial y} > 0. \qquad (5)$$

This condition is a **"second-order condition for a maximum"**. Here, **in an interior maximum, the marginal cost curve is strictly increasing.**

ftbpFU2.904in3.1894in0ptLong-run profit maximizationFigure5Figure

Outline

Long-Run Supply Function

ftbpFU4.0343in2.0332in0ptConstruction of the Firm's Supply Function (Pink Curve with a discontinuity at $p_{\min}$ shown by the dashed part)Figure6Figure

Outline

Short-Run Profit Maximization

- z_2 is a constrained input while z_1 is a variable input.
- Analysis is very similar to that developed for the Long-run profit maximization.
- The difference comes from the fact that the short-run (total) cost function is used instead of the long-run (total) cost function. Let $S(p_1, p_2, \overline{z}_2, y)$ be the short-run (total) cost of producing y units of inputs given input prices p_1 and p_2 given the short-term availability constraint on input 2.

Short-Run Profit Maximization: Optimum

- Firm's problem: choose y to maximize profits $\pi := py - S\left(p_1, p_2, \overline{z}_2, y\right).$
- If $y^* > 0$ is the optimum, then the following necessary condition for a maximum must be satisfied:

$$\frac{\partial \pi}{\partial y} = 0 \Leftrightarrow p = \frac{\partial S\left(p_1, p_2, \overline{z}_2, y\right)}{\partial y}. \qquad (6)$$

Theorem

When the output level is chosen optimally, the cost of producing an extra unit of output in the short run is equal to its price.

- As previously noted, a second-order condition must be checked as well. At the optimum output level, the short term marginal cosgt must be increasing.

Outline

- **Market supply function.** Assume I firms produce a same output: given an output price equal to p, firm i produces $y_i(p)$ units of output $(i = 1, ...I)$. If we call $Y(p)$ the market supply at price p, we have:

$$Y(p) := \sum_{i=1}^{I} y_i(p).$$

- We have already constructed the **demand function** of the market, using the theory of the consumer. If there are N consumers with individual demand functions $x_1(p), ..., x_N(p)$, then the market demand is:

$$D(p) = \sum_{i=1}^{N} x_i(p).$$

- Up to now, the output price p is exogenously given. But, given a price p, we know what is the demand and the supply. The next chapter examines **how the output price p is determined**.