Applied Game Theory
Problem Set 2

Exercise 1

Finitely-repeated simutaneous-move game. The following two-player simultaneously game

1\2	L	C	R
T	3,1	0,0	5,0
M	2,1	1,2	3,1
B	1,2	0,1	4,4

is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting.

Can the payoff $(4,4)$ *be achieved in the first-stage in a pure-strategy subgame-perfect Nash equilibrium? If such is the case, give strategies that do so. Otherwise, prove why not.*

Exercise 2

Tragedy of the commons. Two fishermen have access to a common inshore fishery. Each fisherman ι simultaneously chooses how much effort $x_i \geq 0$ to exert. Let

$$X := x_1 + x_2$$

be the total amount of effort and

$$f(X) = aX - X^2$$

be the total amount of fish extracted. The opportunity cost of effort is ω per unit of effort for each fisherman and the share that fisherman ι gets is x_i/X.

It is assumed that the price of fish is 1 per unit. So, the payoff function of fisherman i writes:

$$u_i(x_1, x_2) = \frac{x_i}{X} f(X) - \omega x_i.$$

Social welfare corresponds to the Benthamite social welfare function:

$$W(x_1, x_2) = \sum_{i=1}^{2} u_i(x_1, x_2).$$

1. Find the Nash equilibrium levels of effort, total fish extracted and the social welfare W.

2. Compare the total extraction level and the social welfare to the socially optimal levels (social optimum occurs when $W(x_1, x_2)$ is maximized). Is there over- or under-fishing relative to the socially optimal level? Comment.

3. Consider now the infinitely repeated version of this game, where in each period $t = 1, ...,$ the fishermen choose simultaneously effort levels (x_1^t, x_2^t) to play the above game. The payoff of fisherman i is given by the average discounted sum of period payoffs, i.e.

$$(1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_i\left(x_1^t, x_2^t\right),$$

   where $0 < \delta < 1$.
   Let the stage game Nash equilibrium effort level be x^e (i.e. what has been found in Question 1) and the socially optimum total effort level be X^o (i.e. what has been found in Question 2). Consider the following grim-trigger strategy profile: choose $X^o/2$ in the first period and after any history in which

both firms have always played $X^o/2$. After any period and after any history in which both firms have always played $X^o/2$; after any other history, choose x^e. In other words, the strategy profile is given by

$$s_i^*\left(h^t\right) = \begin{cases} \frac{X^o}{2}, \ t=1, \\ \frac{X^o}{2}, \ h^t = \left(\underbrace{\left(\frac{X^o}{2}, \frac{X^o}{2}\right), \ldots, \left(\frac{X^o}{2}, \frac{X^o}{2}\right)}_{t-1 \text{ times}} \right), \\ x^e, \ \text{otherwise}, \end{cases}$$

for $i = 1, 2$. For which values of δ, if any, is this strategy profile a subgame perfect equilibrium of this infinitely repeated game?

Exercise 3

Three Oligopolists operate on a market with inverse demand given by $P = a - Q$ where $Q = q_1 + q_2 + q_3$. Each firm has a constant marginal cost c and no fixed cost. The firms choose their quantities as follows:

1. First, Firm 1 chooses $q_1 \geq 0$, which is observed by Firms 2 and 3.
2. Then, Firm 2 and Firm 3 simultaneously choose q_2 and q_3.

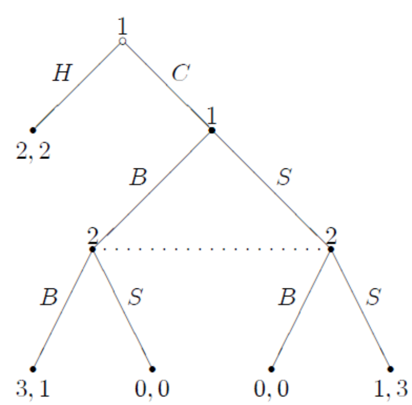
What is the subgame perfect outcome of this game?

Exercise 4

Battle of the Sexes with an Outside Option. Consider the following extensive form game. Player 1 either decides to go to a concert with player 2 in which case they engage in the following Battle of the Sexes game (where B stands for Bach and S for Stravinsky Concert):

1\2	B	S
B	3,1	0,0
S	0,0	1,3

or he chooses to stay Home in which case both players receive a payoff of 2. This results in the extensive form game (where H stands for Home and C for Concert):



1. Write down the strategic form of this game and find all its pure strategy Nash equilibria.
2. Find the set of pure strategy subgame perfect equilibria of this game.