Applied Game Theory
Problem Set 1

Exercise 1

Consider the strategic form game represented by the following bimatrix, where player A is the row and player B is the column player:

1\2	L	R
T	a, b	$c, 2$
M	$1, 1$	$1, 0$
B	$3, 2$	$0, 1$

a, b and c are numbers which are left unspecified for now.

1. For which values of a, b and c, the outcome (T, L) is a strictly dominant strategy equilibrium?
2. For which values of a, b and c, the outcome (T, L) is a pure-strategy Nash equilibrium.

Exercise 2

Consider the inspection game described by the following bimatrix:

1\2	Inspect	Nap
Work	$2, 2$	$2, 3$
Shirk	$1, 4$	$3, 2$

Represent graphically the best-response correspondences and find the mixed-strategy Nash equilibrium of the game.

Exercise 3

Two lumberjacks ($i = 1, 2$) exploit a same forest to produce planks. The more the common resource is used, the less planks any given individual can produce.

The amount of the common resource used by individual i is denoted x_i. More specifically, individual i's output is

$$\begin{cases} x_i\,(1-(x_1+x_2)) & \text{if } x_1+x_2 \leq 1, \\ 0 \text{ otherwise.} \end{cases}$$

Each individual i chooses $x_i \in [0, 1]$ in order to maximize his output.

1. Formulate this situation as a strategic game.
2. Find the best response correspondences of the players.
3. Find the Nash equilibria of the game.
4. Does the Nash equilibrium value x_1, x_2 maximize the total output?

Exercise 4

We now consider a model based on Bertrand's suggestion that firms actually choose prices rather than quantities. Here, we consider the case of differentiated products.

Two firms ($i = 1, 2$) choose prices p_1 and p_2, respectively. The quantity that consumers demand from firm i is:

$$q_i\left(p_i, p_j\right) = a - p_i + b p_j,$$

where $b > 0$ reflects the extent to which firm i's product is a substitute for firm j's product. There are no fixed costs of production and marginal costs are constant at c, where $0 < c < a$. The payoff of firm i is simply its profit. Both firms choose their prices simultaneously.

1. Write down the normal form of the game, given that negative prices are not feasible but any non-negative price can be charged.

2. Find the Nash equilibrium.