3. The Deadweight Loss of Taxation

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INTRODUCTION

The efficiency costs associated with taxation

Government raises taxes for one of two reasons:

1. To raise revenue to finance public goods
2. To redistribute income

But to generate 1 of revenue, welfare of those taxed falls by more than 1 because the tax distorts behavior

How to implement policies that minimize these efficiency costs?

$\Longrightarrow$ Start with positive analysis of how to measure efficiency cost of a given tax system

THE AVERAGE DEADWEIGHT LOSS OF INCOME TAXATION

The deadweight loss of income taxation

Why does an income tax create distortions? Because it creates **a wedge between the private and social benefits** of additional work.

- The **private benefits** of additional work consists of the net-of-tax wage to the worker
- The **social benefits** of additional work consists of the net-of-tax wage to the worker **and** the additional tax revenue for the government

As long as the income tax $t > 0$, the social benefits of work $>$ the private benefits of work $\Longrightarrow$ people supply too little labor

The total distortive cost of the income tax is measured by its **deadweight loss**:

How much more revenue would the government have if it were to replace the distortive income tax by a non-distortive lump-sum tax, such that all individuals are perfectly compensated?

Basic assumptions

To illustrate the total/average deadweight loss of taxation, we focus on the simplest possible case

Firms are competitive, production is linear in labor supply nl, with n productivity per unit of labor

$\Longrightarrow$ before-tax wages equal $w = n$, labor demand perfectly elastic

Individuals maximize quasi-linear utility:

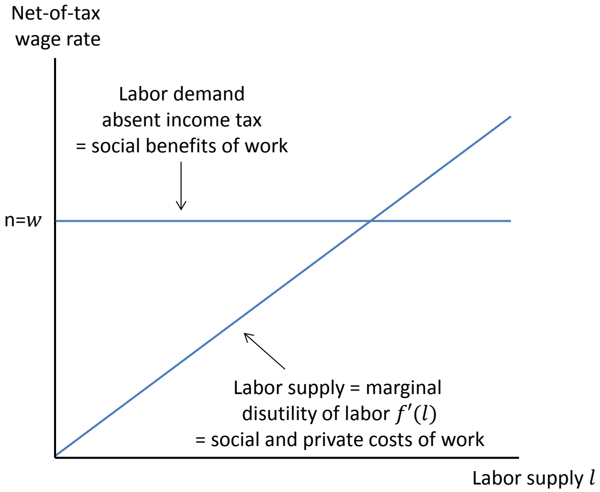
$$U = c - f(l)$$

subject to the budget constraint $c = (1 - t)wl - T$

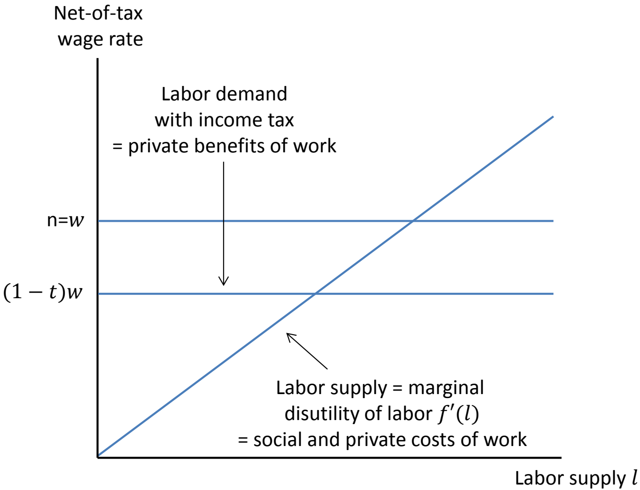
FOC: $f'(l) = (1 - t)w \Longrightarrow$ labor supply only a function of the income tax: $l^* = l(t)$ (no income effects)

Furthermore assume that disutility is quadratic in labor supply $f(l) = \alpha l^2$, such that marginal disutility of work $f'(l)$ is linear in labor supply $f'(l) = 2\alpha l$

The dead-weight loss of income taxation



The dead-weight loss of income taxation



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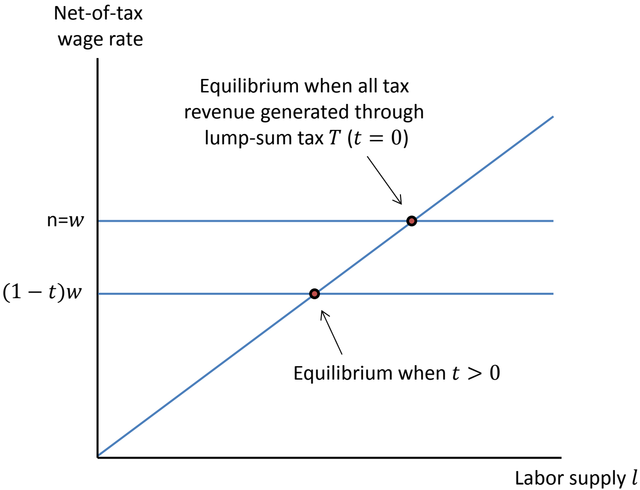

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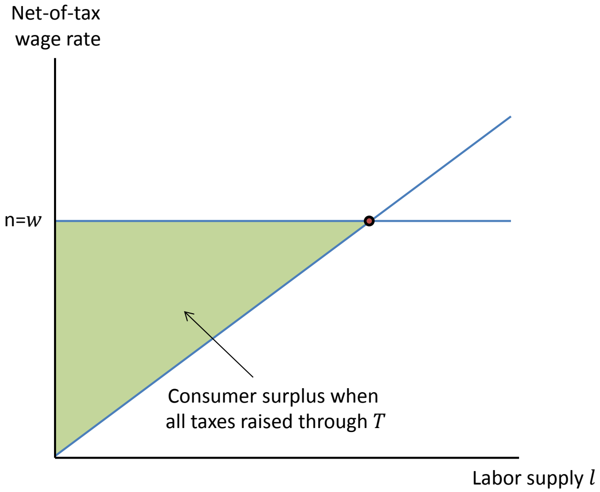

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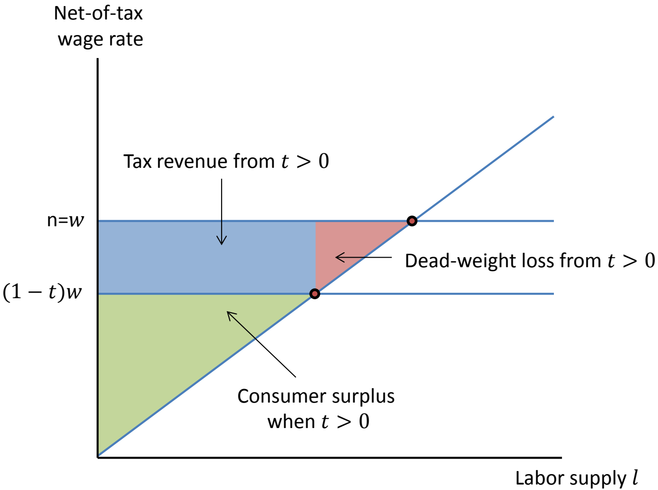
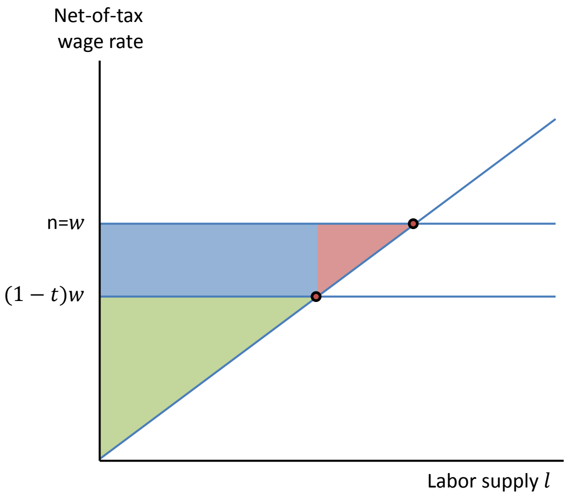


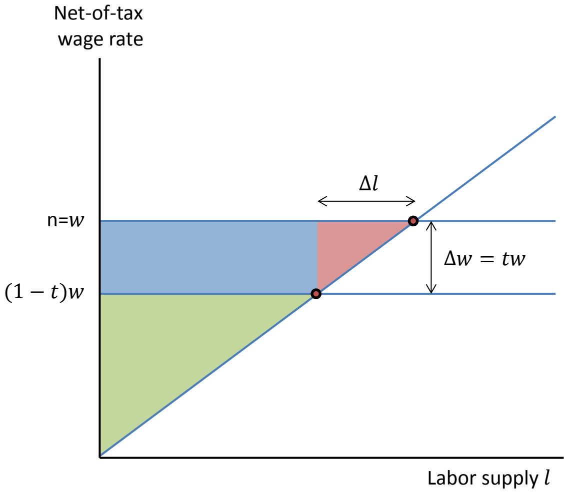
The dead-weight loss of income taxation



Total DWL given by the surface of the red triangle:

$$DWL = \frac{1}{2} * base * height$$

The dead-weight loss of income taxation



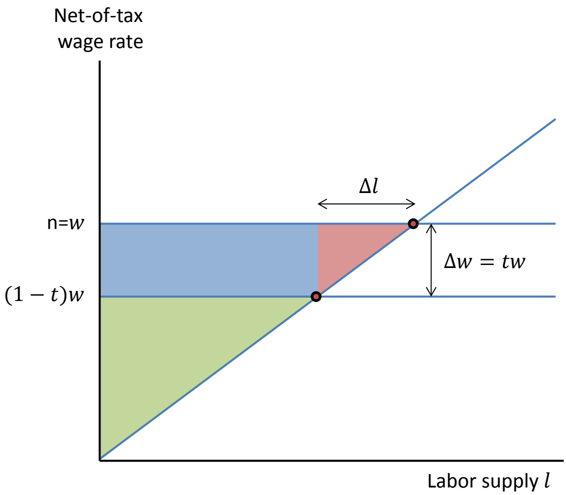
Total DWL given by the surface of the red triangle:

$$DWL = \frac{1}{2} * base * height$$

Base equals Δl; height equals $\Delta w = tw$:

$$DWL = \frac{1}{2}\Delta l tw = \frac{1}{2}\frac{\Delta l}{\Delta w}\frac{w}{l}t^2 twl$$

The dead-weight loss of income taxation



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Wage elasticity of labor supply given by $e \equiv \frac{\Delta l}{\Delta w}\frac{w}{l}$

Average dead-weight loss, or DWL per € of total income:

$$ADWL \equiv \frac{DWL}{wl} = \frac{1}{2}t^2 e$$

The average deadweight loss of income taxation

The average deadweight loss of income taxation gives the amount of resources wasted as a share of total income (waste/GDP):

$$\boxed{ADWL = \frac{1}{2}t^2 e}$$

We can learn two things from this expression:

- The average deadweight loss of income taxation increases with the square of the tax rate [implications?]
- The average deadweight loss of income taxation is proportionally increasing in the elasticity [implications?]

Illustration with $e = 0.3$ and $t = 0.56$ suggests that $ADWL \approx 0.05 \implies$ roughly 5 percent of total income is wasted due to income-tax distortions!

The exact number for the $ADWL$ is to be taken with a grain of salt, however, as there are a number of important caveats:

- The labor supply curve might not be linear (i.e., $f(l)$ not quadratic)
- There is more than one income tax rate ($\Longrightarrow$ not just one DWL measure, but one for every different tax rate)
- Labor demand may not be perfectly inelastic

Partly because of the caveats associated with the $ADWL$, we are often more interested in the marginal deadweight loss: what is the efficiency loss of raising an additional unit of tax revenue through the income tax?

THE MARGINAL DEADWEIGHT LOSS OF INCOME TAXATION

The marginal deadweight loss of income taxation

For actual policy reforms, the more relevant measure of the distortive costs of taxation is the **marginal deadweight loss**:

What is the tax-revenue loss associated with raising an additional unit of income taxes, while compensating individuals through a reduction in the lump-sum tax?

Tax reform leaves individual utility unaffected but weakens incentives to supply labor $\Longrightarrow$ compensated labor supply reduction $\Longrightarrow$ loss in tax revenue (distortive costs also referred to as **tax base erosion**)

The marginal deadweight loss gives a measure of the resource waste associated with raising income taxes rather than lump-sum taxes

Because the income tax is more progressive than a lump-sum tax, the marginal deadweight loss is also sometimes called **the price of equality**

A more general model

A representative individual supplies l units of labor at a gross wage w, pays an income tax at rate t and a lump-sum tax T:

$$c = (1 - t)wl - T$$

Sets l to maximize utility:

$$U = u(c, l) = u((1 - t)wl - T, l)$$

FOC: $-u_l = (1 - t)wu_c$, yields labor supply $l^* = l(t, T)$

We will consider a marginal increase in the income tax $\mathrm{d}t > 0$, and a reduction in the lump-sum tax $\mathrm{d}T < 0$, such that the individual is perfectly compensated, $\mathrm{d}U = 0$

What is the reform's effect on the individual's labor supply?

Taking the total derivative of $l^* = l(t,T)$ to obtain the labor supply response to the reform:

$$\mathrm{d}l^* = \frac{\partial l}{\partial t}\mathrm{d}t + \frac{\partial l}{\partial T}\mathrm{d}T = \frac{\partial l}{\partial t}\mathrm{d}t - \frac{\partial l}{\partial T}wl^*\mathrm{d}t$$

where we substituted for $\mathrm{d}T = -wl^*\mathrm{d}t$ in the second equation. Dividing by $\mathrm{d}t$ yields:

$$\frac{\mathrm{d}l^*}{\mathrm{d}t} = \frac{\partial l}{\partial t} - \frac{\partial l}{\partial T}wl = \frac{\partial l^c}{\partial t} < 0$$

where $\frac{\partial l^c}{\partial t}$ gives the **compensated** labor supply response [why?]. We obtain Slutsky's decomposition (Lecture 1).

Total **government revenue** from the income tax and the lump-sum tax equals:

$$\mathcal{R} = twl^* + T$$

Recall the definition of the marginal DWL: *the tax-revenue loss associated with raising an additional unit of income taxes, while compensating individuals through a reduction in the lump-sum tax*.

An increase in the income tax yields $wl^*\mathrm{d}t$ additional tax revenue:

$$MDWL \equiv -\frac{\mathrm{d}\mathcal{R}}{wl^*\mathrm{d}t}\bigg|_{\mathrm{d}T=-wl^*\mathrm{d}t}$$

Take total derivative of $\mathcal{R}$ to get:

$$\mathrm{d}\mathcal{R} = tw\mathrm{d}l^* + wl^*\mathrm{d}t + \mathrm{d}T$$

Substitute for $\mathrm{d}T = -wl^*\mathrm{d}t$ and for $\mathrm{d}l^* = \frac{\partial l^c}{\partial t}\mathrm{d}t$ (from the previous slide):

$$\mathrm{d}\mathcal{R} = tw\frac{\partial l^c}{\partial t}\mathrm{d}t$$

Substituting back into the definition of the marginal deadweight loss yields:

$$MDWL = -\frac{tw\frac{\partial l^c}{\partial t}\mathrm{d}t}{wl^*\mathrm{d}t} = \frac{t}{1-t}\left(-\frac{\partial l^c}{\partial t}\frac{1-t}{l^*}\right)$$

Finally, we can once more define the **compensated net-of-tax rate elasticity of labor supply** as:

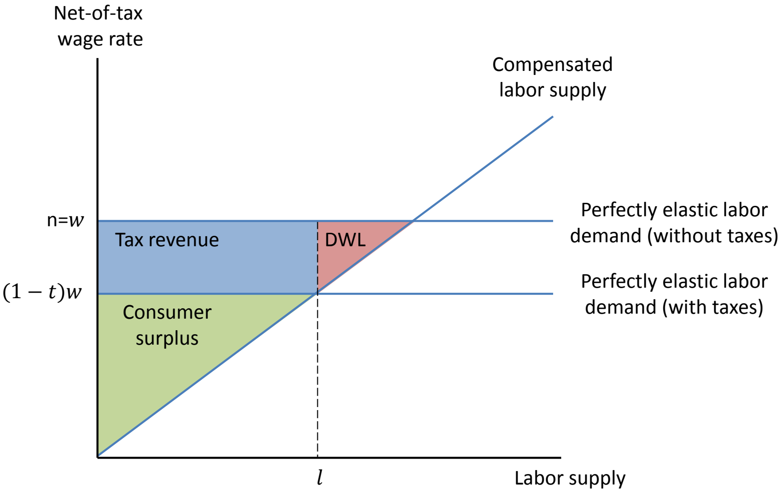
$$e \equiv -\frac{\partial l^c}{\partial t}\frac{1-t}{l^*}$$

This then yields the marginal deadweight loss of income taxation:

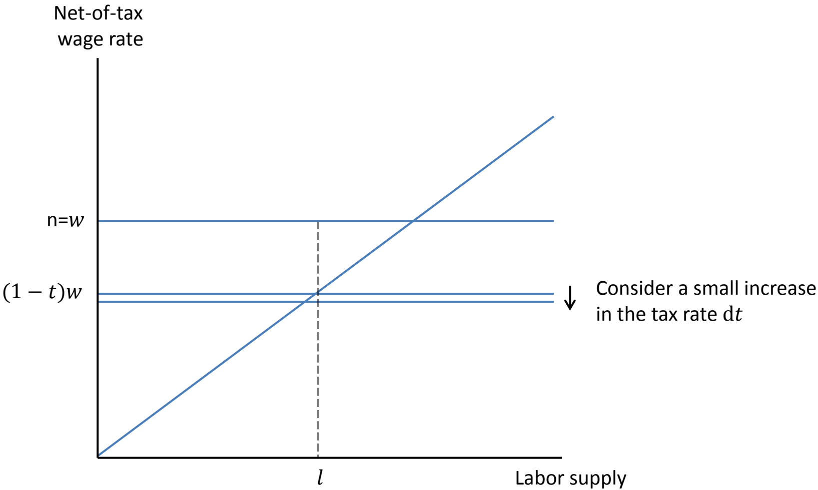
$$MDWL = \frac{t}{1-t}e$$

See next slides for a graphical derivation $\Longrightarrow$

Marginal dead-weight loss of income taxation



Marginal dead-weight loss of income taxation



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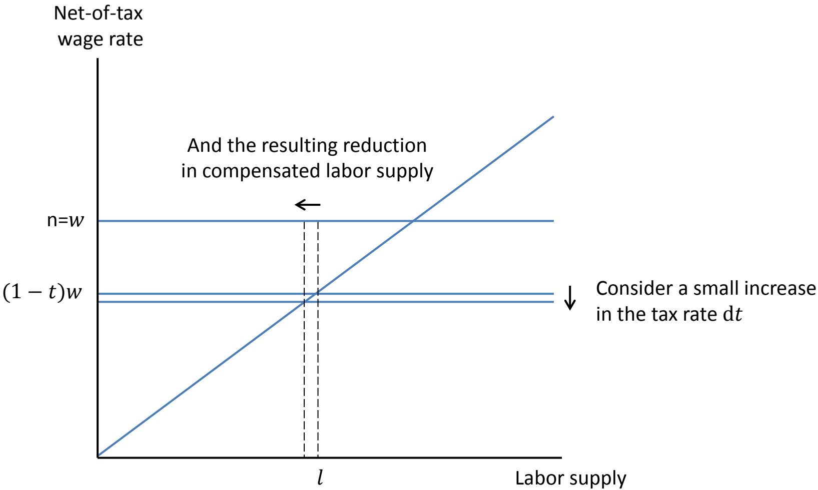

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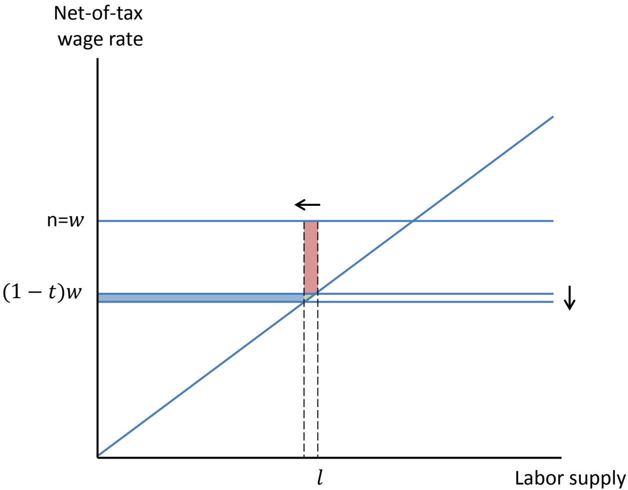

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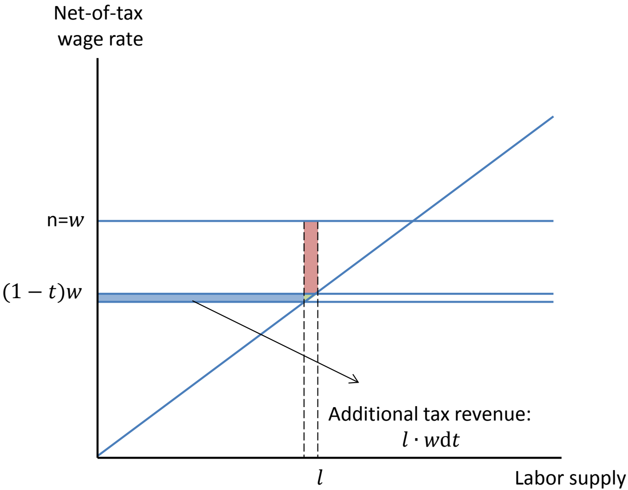

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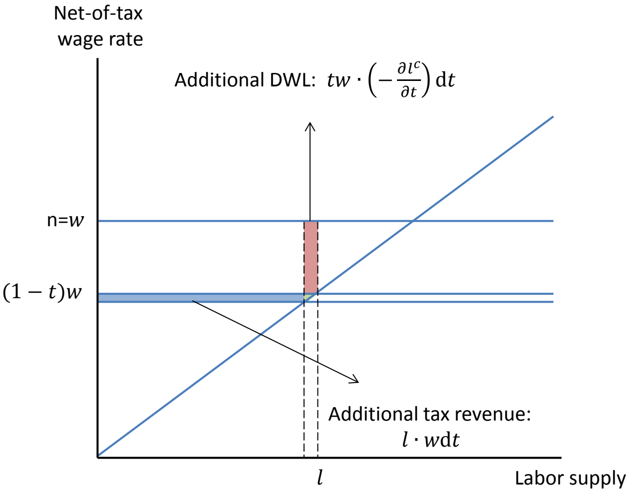
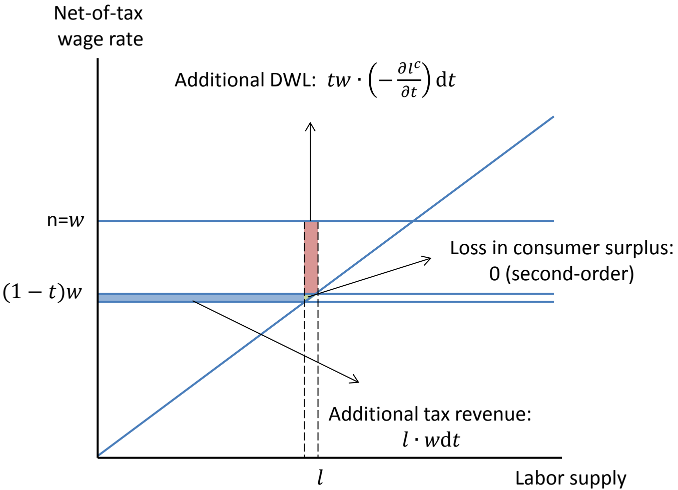
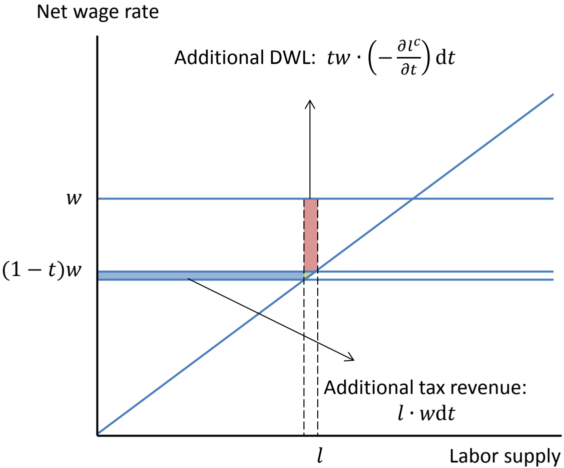


Marginal dead-weight loss of income taxation



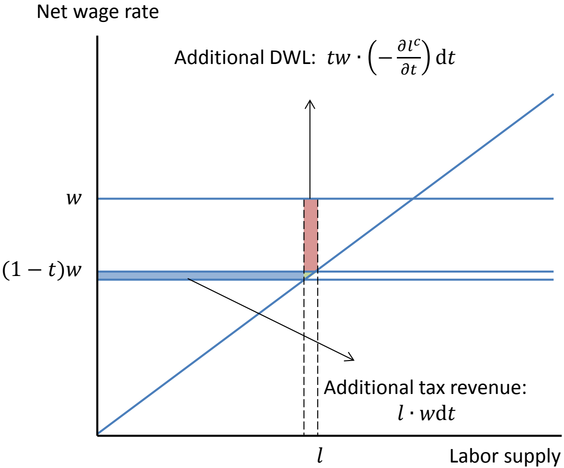
Marginal dead-weight loss of income taxation



Marginal dead-weight loss equals additional DWL per additional unit of tax revenue:

$$MDWL = \frac{t}{1-t} \cdot \left(-\frac{\partial l^c}{\partial t} \frac{1-t}{l}\right)$$

Marginal dead-weight loss of income taxation



Marginal dead-weight loss equals additional DWL per additional unit of tax revenue:

$$MDWL = \frac{t}{1-t} \cdot \left(-\frac{\partial l^c}{\partial t}\frac{1-t}{l}\right)$$

Define the compensated net-of-tax rate elasticity of labor supply as:

$$e = -\frac{\partial l^c}{\partial t}\frac{1-t}{l}$$

Thus:

$$MDWL = \frac{t}{1-t} \cdot e$$

The marginal deadweight loss of income taxation

Thus, the marginal deadweight loss of income taxation is given by:

$$\boxed{MDWL = \frac{t}{1-t}e}$$

The marginal deadweight loss is increasing in:

- the tax rate $\Longrightarrow$ the larger t, the larger the revenue losses from a decrease in l
- the elasticity $\Longrightarrow$ the larger e, the stronger the labor supply response to an increase in taxes

Furthermore, notice that $MDWL = 0$ if $t = 0$ [**WHY?**]

A back-on-the-envelope calculation with $e = 0.3$ and $t = 0.56$ suggests that $MDWL \approx 0.38 \Longrightarrow$ roughly 38 cents are wasted for every additional income tax revenue!

Caveats

We made a number of assumptions that may affect our estimates for the marginal deadweight loss of income taxation:

- This is just the average $MDWL$, but every tax bracket might have different t and different $e \Longrightarrow MDWL$ may be different for each bracket. Intuitively, one would expect higher $MDWL$ for higher tax brackets because high-income individuals face higher marginal taxes and higher elasticities.
- Labor demand may be imperfectly elastic, in which case the marginal deadweight loss equals $MDWL = \frac{t}{1-t}\left(\frac{e^d}{e+e^d}\right)e$, with e^d the labor demand elasticity. However, it does not make much difference for realistic values of e^d.
- When wages are uncertain, t taxes the lucky more than the unlucky and thus partially corrects for a missing insurance market. In that case also efficiency *gains* associated with income tax.

- The labor market might already be distorted by imperfect competition (e.g., union monopoly, employer monopsony). Taxes might then also worsen/improve pre-existing distortions.
- There might be some administrative costs associated with tax collection, but the estimates that we have suggest they are probably very small on the margin.
- Workers may be involved in a rat race and derive utility from **relative income** rather than absolute income. Thus, they create negative external effects on others by working more. In that case, taxes might also lead to efficiency gains by reducing labor supply.

Exercise: Cobb-Douglas utility function [do at home]

Assume that a representative agent supplies a share l of his time on labor and thus a share $1 - l$ on leisure activities. His utility function can be written as $u(c, l)$. There is a labor-income tax t and a lump-sum tax T. Wages are normalized to $w = 1$ so that the budget constraint is given by $c = (1 - t)l - T$.

1. In the Lectures, we proved that a necessary condition for equilibrium labor supply is $-u_l = (1 - t)u_c$, where $u_l \equiv \frac{\partial u}{\partial l}$ and $u_c \equiv \frac{\partial u}{\partial c}$. Interpret this condition in terms of marginal costs and benefits.
2. Assume that $u(c, l) = c^{\alpha}(1 - l)^{1-\alpha}$. This is also called a **Cobb-Douglas** utility function. Show that the condition for equilibrium labor supply can be written as $\frac{1-\alpha}{\alpha}\frac{c}{1-l} = 1 - t$.
3. Use this equation and the budget constraint to obtain equilibrium labor supply as a function of taxes: $l^* = l(t, T) = \alpha + (1 - \alpha)\frac{T}{1-t}$.
4. Show that the uncompensated net-of-tax rate elasticity of labor supply is given by:

$$e^u \equiv -\frac{\partial l}{\partial t}\frac{1 - t}{l} = -\frac{(1 - \alpha)\frac{T}{1-t}}{\alpha + (1 - \alpha)\frac{T}{1-t}}$$

.

5. Prove that if the lump-sum tax is zero, the uncompensated labor-supply elasticity is $e^u = 0$. Explain how this is possible?
6. Explain why you cannot conclude that income taxation is not distortive if the uncompensated labor-supply elasticity is zero.

MEASURING DEADWEIGHT LOSS

Measuring effective marginal taxes

To calibrate $MDWL = \frac{t}{1-t}e = 0.38$ we relied on two different estimates:

- we assumed that the effective marginal tax rate equals $t = 0.56$
- and we assumed that the elasticity equals $e = 0.3$

But where do these estimates come from?

In reality, labor supply distorted by both income taxes $\tilde{t}$ and an effective commodity tax rate τ

Recall from Lecture 1 that the effective net-of-tax rate can be written as $1 - t = \frac{1-\tilde{t}}{1+\tau}$, such that $t = \frac{\tilde{t}+\tau}{1+\tau}$

In a country like France or the NL, the average effective tax on income is estimated at $\tilde{t} = 0.50$; the average effective commodity tax is estimated at $\tau = 0.123 \implies$ this yields $t \approx 0.56$

Measuring elasticities

Measuring net-of-tax rate elasticities is harder, mainly due to two issues:

1. how do we measure l? is it labor hours? labor effort? how about other types of tax base erosion?
2. and how do we measure the effect of $(1-t)$ on l?

Issue 1 solved by Feldstein (1995, 1999): we do not need to distinguish between all kinds of behavioral responses that reduce (taxable) labor income; instead, we can simply focus on taxable income $z \equiv wl$ and write the elasticity as:

$$e \equiv \frac{\mathrm{d}l}{\mathrm{d}(1-t)} \frac{1-t}{l} = \frac{\mathrm{d}z}{\mathrm{d}(1-t)} \frac{1-t}{z}$$

This brings two important advantages:

- the elasticity of taxable income takes into account all avoidance/evasion behavior that reduces the tax base
- individual data on z is readily available from the tax authority

Second issue is more difficult: imagine that the government raises tax t in year y.

Then we could measure changes in individuals' incomes $\Delta z_y = z_y - z_{y-1}$ and changes in net-of-tax rates $\Delta(1 - t_y) = -\Delta t_y = -t_y + t_{y-1}$ and estimate:

$$\hat{e} = \frac{\Delta z_y}{\Delta(1 - t_y)} \frac{1 - t_{y-1}}{z_{y-1}} = \frac{\text{\% change } z_y}{\text{\% change } (1 - t_y)}$$

But this is problematic: maybe income changed for unrelated reasons (e.g., business cycle).

Instead, we need a **counterfactual**: what would % change z_y have been if the individuals had **not** received a tax increase?

For this reason, good empirical studies pay a lot of attention to identifying individuals who did not receive the tax increase but are very similar to the ones who did.

For example, imagine that the government raises taxes in the second bracket but not the first.

In other words, tax payers in the second bracket are the **treatment group** and tax payers in the first bracket are the **control group**

We can then control for changes in z that would have occurred without the tax increase by using a **difference-in-difference** methodology:

$$\hat{e} = \frac{\%\text{ change } z_y^{\text{treatment}} - \%\text{ change } z_y^{\text{control}}}{\%\text{ change } (1 - t_y^{\text{treatment}}) - \%\text{ change } (1 - t_y^{\text{control}})}$$

$\Longrightarrow$ numerator gives % change in z *vis-a-vis* the control group

$\Longrightarrow$ denominator gives % change in $(1 - t)$ *vis-a-vis* the control group

Also see example next page $\Longrightarrow$

Illustration: do at home!

Example: imagine that at the beginning of 2016, the government raises the tax rate in the second bracket from 40% to 46%, while keeping the tax rate in the first bracket constant at 30%

Furthermore, average income in the first bracket increased from 20 in 2015 to 25 in 2016; average income in the second bracket stagnated at 40 in both 2015 and 2016

Fill in the table below:

income	2015	2016	% change
bracket 1			
bracket 2			
diff-in-diff			

net-of-tax rate	2015	2016	% change
bracket 1			
bracket 2			
diff-in-diff			

Calculate the estimated net-of-tax rate elasticity of taxable income:

$$\hat{e} = \frac{\text{\% change } z_y^{\text{treatment}} - \text{\% change } z_y^{\text{control}}}{\text{\% change } (1 - t_y^{\text{treatment}}) - \text{\% change } (1 - t_y^{\text{control}})}$$

Compare with what you would have found had you not taken the difference-in-difference approach. Interpret.

Measuring elasticities

All modern approaches to estimating elasticities are based on either a difference-in-difference methodology or some other ingenuous method to creating a proper counterfactual.

See the review of Saez, Slemrod and Giertz (2012) for international estimates.

From this literature, estimates of $\hat{e} = 0.3$ seem to be highly reasonable.

SUMMING UP

Key insights from the Lecture

- Taxation creates distortions because it drives a wedge between the social and the private benefits of economic activity
- The deadweight loss of income taxation measures the efficiency loss of using the income tax as compared to using a non-distortive lump-sum tax
- The marginal deadweight loss gives the loss in tax revenue due to a compensated increase in the income tax
- The marginal deadweight loss is increasing in the tax rate and the compensated net-of-tax rate elasticity of taxable income
- If some individuals are affected by a change in tax rates while other similar individuals are not, we can use a difference-in-difference methodology to estimate the elasticity of taxable income

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