The objectives of the producer

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Contents

1. MINIMIZING COSTS

A firm uses inputs in order to produce outputs. Its objective is not always to maximize profits, but whatever its objective, a rational firm must minimize the production costs. Hence, given the chosen output level (the choice of the output level is addressed in section 2), the firm chooses the input combination for which the production cost is minimum.

Time is an important aspect in the firm's decision to produce a certain amount of output. It is possible to distinguished **two kinds of inputs**:

- **variables inputs**: when an input is variable, the amount used by the firm can be varied at will by the firm.
- **constrained inputs**: it takes at least period of time to vary the amount of constrained inputs used by the firm.

In the short run, the firm usually employs a combination of variable and constrained inputs. For example, labour is usually more flexible as capital; hence, labour may be regarded as a variable input and capital as a constrained input. The implication is that – in the short term – the firm can only decide about the amounts of variable inputs it uses whilst the amounts of constrained inputs are fixed.

In the long run, all inputs are variables. Hence, the firm does not face the same constraints as in the short run and can choose any input combination.

The long run is considered first because, in that case, there is no constraint on the amount of inputs the firm can employ. The short run is then examined and the links between the short run and long run are finally emphasized.

1.1. Long-Run Cost Minimization

For convenience, it is considered that there are **only two inputs**, labelled 1 and 2. Let p_i be the (exogenously) given price of one unit of input i ($i = 1, 2$). The firm uses quantities z_1 and z_2 of these inputs, respectively, in order to produce q units of output. As previously noted, q is a given number (whose determination is examined in Section 2 below).

Because we consider the long-run, both z_1 and z_2 are free to vary, without any constraint. Hence, the problem of the firm can be written as follows:

Problem 1 (Long-Run Cost Minimization) *Choose the input combination* (z_1, z_2), *to produce output* q, *which minimizes the production cost* $p_1 z_1 + p_2 z_2$.

1.1.1. Graphical Solution

It is easy to solve this problem graphically. First, we need to introduce the production function $f(z_1, z_2)$ which specifies the maximum amount of output that the firm can produce when using z_1 units of input 1 and z_2 units of input 2. We know that the firm wants to produce q units of output. The **isoquant of level** q, denoted by IS_q, represents all the input combinations (z_1, z_2) for which output $f(z_1, z_2)$ is equal to q or, formally,

$$IS_q := \left\{(z_1, z_2) \in \mathbb{R}_+^2 : f(z_1, z_2) = q\right\}.$$

The production function is assumed to be twice continuously differentiable and the marginal productivities of input 1 and input 2 are decreasing. In that case, the isoquant IS_q is "smooth" and decreasing in the (z_1, z_2)-space.

Isocost curves are the second ingredient in order to solve the problem graphically ("iso" means equal in Greek). By definition, the isocost curve of level c, denoted C, represents all input combinations whose cost is c. Formally,

$$\left\{(z_1, z_2) \in \mathbb{R}_+^2 : p_1 z_1 + p_2 z_2 = C\right\}.$$

Here, p_1 and p_2 do not depend on the quantities of inputs bought by the firm. Hence, the isocost curves are actually isocost lines, with equation:

$$z_2 = \frac{C}{p_2} - \frac{p_1}{p_2} z_1.$$

There are an infinity of isocosts lines in the (z_1, z_2)-space, with cost C ranging from 0 to infinity.

The **solution** to Problem 1 is shown in Figure 1. The minimizing input combination (z_1^*, z_2^*) is obtained at the tangency point between the isoquant IS_q and

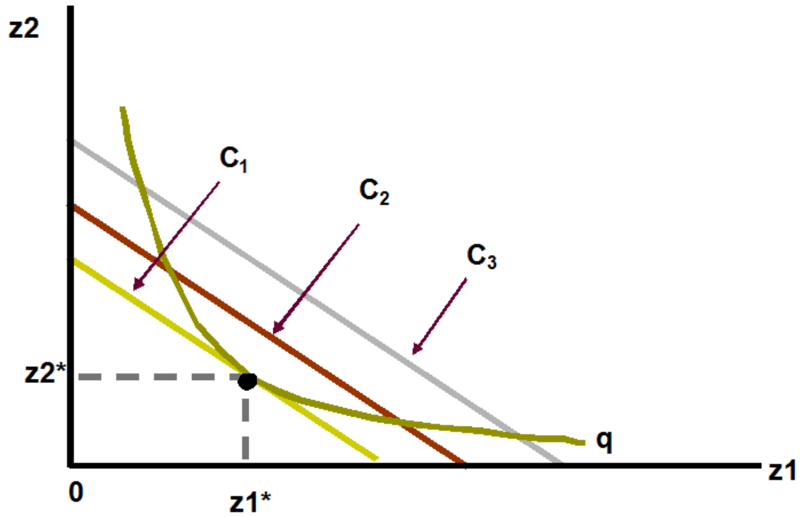

Figure 1: Long-run cost minimization. The cost-minimizing input combination is obtained at the tangency point between the isoquant of level q (q is exogenously given) and the lowest indifference curve (here C_1).

the lowest isocost curve. The minimum cost to produce q units of output is thus $p_1 z_1^* + p_2 z_2^*$. The following proposition characterizes the optimum input combination.

Proposition 1 (Cost-Minimizing Input Combination) *Assume z_1 and z_2 are both strictly positive in the cost-minimizing input combination. Then, costs are minimized at the input combination (z_1^*, z_2^*) which satisfies*

$$MRTS_{12}(z_1^*, z_2^*) = \frac{p_1}{p_2}. \quad (1)$$

Remark 1 (Conditional Input Demands) *Note that (z_1^*, z_2^*) are functions of the input prices p_1 and p_2 and of the output level q. For this reason, we can use the following notations: $z_1^*(p_1, p_2, q)$ and $z_2^*(p_1, p_2, q)$. $z_1^*(p_1, p_2, q)$ and $z_2^*(p_1, p_2, q)$ are called* conditional input demands *because they specify which amounts of inputs must be used conditionally to the fact that the firm wants to produce q units of output.*

Because the marginal rate of technical substitution $MRTS_{12}(z_1, z_2)$ is also equal (by definition) to the ratio of the marginal products of inputs 1 and 2, i.e.,

$$MRTS_{12}(z_1, z_2) = \frac{\partial f(z_1, z_2)/\partial z_1}{\partial f(z_1, z_2)/\partial z_2},$$

the characterization of the optimum (1) can be rewritten as:

$$\frac{\partial f(z_1^*, z_2^*)/\partial z_1}{\partial f(z_1^*, z_2^*)/\partial z_2} = \frac{p_1}{p_2} \Longleftrightarrow \frac{\partial f(z_1^*, z_2^*)/\partial z_1}{p_1} = \frac{\partial f(z_1^*, z_2^*)/\partial z_2}{p_2}.$$

Interpretation. In the cost-minimizing input combination, the "marginal products in value" $\frac{\partial f(z_1,z_2)/\partial z_1}{p_1}$ and $\frac{\partial f(z_1,z_2)/\partial z_2}{p_2}$ are equal. To see why, assume this is not the case and, without loss of generality, suppose $\frac{\partial f(z_1,z_2)/\partial z_1}{p_1} > \frac{\partial f(z_1,z_2)/\partial z_2}{p_2}$. If the firm spends one extra euro in input 1, its output level is approximately increased by $\partial f(z_1, z_2)/\partial z_1$ units. If its spends one extra euro in input 2, its output level is approximately increased by $\partial f(z_1, z_2)/\partial z_2$ units only. Hence, the firm has an incentive to further substitute input 1 to input 2 because this allows it to produce the same output while reducing production cost. Consequently, the marginal products in value must be equal at the cost-minimizing input combination.

1.1.2. Lagrange Method

We want to find the solution to the following problem:

$$\min_{z_1 \geq 0, z_2 \geq 0} p_1 z_1 + p_2 z_2 \text{ such that. } f(z_1, z_2) = q.$$

The **Lagrangian** associated with this minimization problem is:

$$L = p_1 z_1 + p_2 z_2 + \lambda [f(z_1, z_2) - q].$$

The **necessary conditions** that must be satisfied by the cost-minimizing (z_1, z_2) are:

$$\frac{\partial L}{\partial z_1} = p_1 + \lambda \frac{\partial f(z_1, z_2)}{\partial z_1} = 0, \quad (2)$$

$$\frac{\partial L}{\partial z_2} = p_2 + \lambda \frac{\partial f(z_1, z_2)}{\partial z_2} = 0. \quad (3)$$

Using (2), one gets:

$$\lambda = -\frac{p_1}{\partial f(z_1, z_2)/\partial z_1},$$

which is substituted in (3) to get:

$$\frac{\partial f(z_1, z_2)/\partial z_1}{p_1} = \frac{\partial f(z_1, z_2)/\partial z_2}{p_2}.$$

1.1.3. Comparative Statics of the Cost-Minimizing Solution

Changing input prices Assume first that **both inputs' prices change in the same proportion**. Then, the change has no effect on the choice of the cost-minimizing input combination because the slope of the isocost lines, equal to $-p_1/p_2$, is not modified.

We now turn to the **impact of increasing the price of an input (or of both inputs but in different proportions)**, all other things being equal. The slope of the isocost curves is then modified. This is shown in Figure 2. Here, p_1 is increased whilst p_2 is decreased. We note that the increase in p_1 induces a *own price input substitution effect*: input 2 is substituted to input 1. At the same time, the decrease in p_2 induces another *own price input substitution effect*: input 2 is substituted to input 1. In each case, the own price input substitution effect is non-positive: an increase in p_i induces a reduction in z_1 in the cost-minimizing input combination.

Changing output level We examine the impact of **increasing output level**, all other things being equal. Assume $q_2 > q_1$. Then, in order to produce q_2 the firm must spend more than for producing q_1. The total cost of production is thus increased: $C(p_1, p_2, q_1) < C(p_1, p_2, q_2)$. This is illustrated in Figure 3. The locus of cost-minimizing input combinations for the different possible values of output is called the **"expansion path" of the firm**.

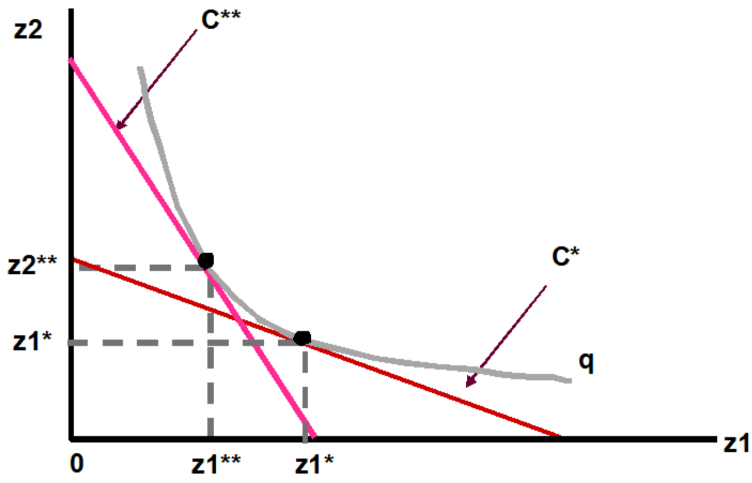

Figure 2: Impact of a Change in Input Prices

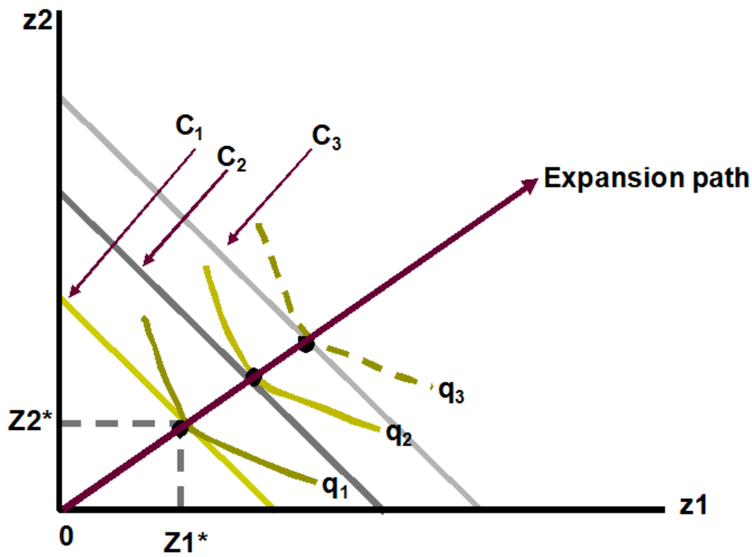

Figure 3: Impact of a Variation in the Output Level q on the Cost-Minimizing Input Combinations.

1.1.4. Cost Functions

Definition 1 (Long-Run Cost Function) *The long-run cost function of the firm is defined has*

$$C(p_1, p_2, q) = p_1 z_1^* + p_2 z_2^*.$$

Hence, given input prices p_1 and p_2 and output level q, the firm's long-run cost function gives the cost of the cheapest input combination which allows the firm to produce q *under the condition that all inputs are variables.* It is clear that:

$$\begin{aligned} \frac{\partial C(p_1, p_2, q)}{\partial p_1} &> 0, \\ \frac{\partial C(p_1, p_2, q)}{\partial p_2} &> 0, \\ \frac{\partial C(p_1, p_2, q)}{\partial q} &> 0. \end{aligned}$$

When the price of an input goes up or when the firm wants to produce more – everything else being constant –, then it must spend more.

The graph of the **long-run cost function** can be obtained from the expansion path (Figure 3). Indeed, the expansion path provides us with the minimum cost to produce output q_1, q_2, q_3, etc. Then, to get the **long-run average cost function**, we can proceed as follows. By definition, the average cost of producing output q is equal to

$$AC(p_1, p_2, q) := \frac{c(p_1, p_2, q)}{q}.$$

Consider a ray through the origin and (q_1, C_1) in Figure 4 (Top Panel). The slope of this ray is C_1/q_1, which is equal to $AC(p_1, p_2, q_1)$. The corresponding average cost for an output level equal to q_1 is shown in Figure 4 (Bottom Panel). To marginal cost is simply the graph of the slope of the total cost function. The different costs are shown in Figure 4. Note very carefully that the marginal cost intersects the average cost at its minimum (here from below).

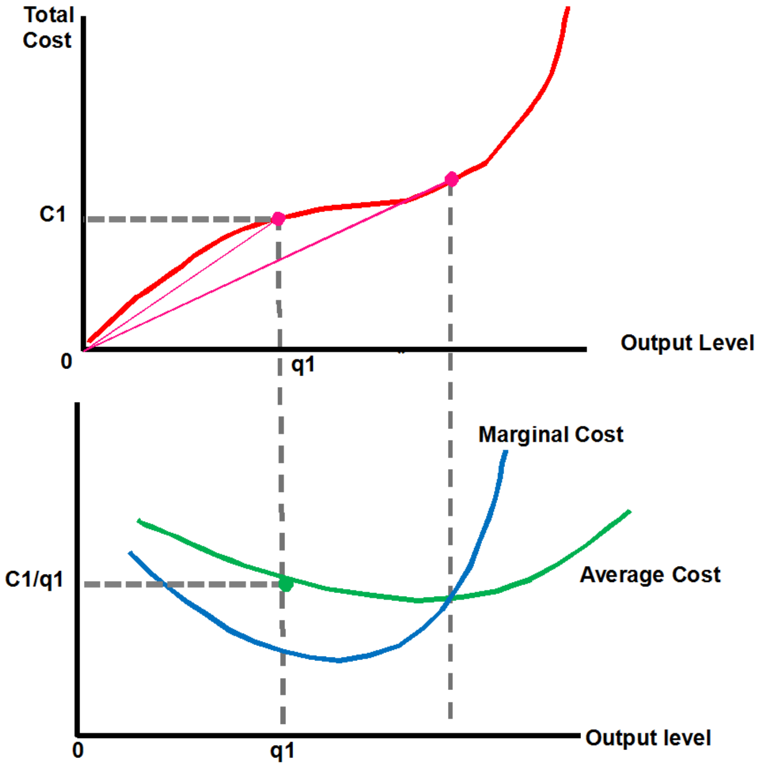

Figure 4: Long-run Cost Functions: Total Cost, Average Cost, Marginal Cost

1.2. Short-run cost minimization

For convenience, it is considered that there are **only two inputs**, labelled 1 and 2. Let p_i be the (exogenously) given price of one unit of input i ($i = 1, 2$). The firm uses quantities z_1 and z_2 of these inputs, respectively, in order to produce q units of output. As previously noted, q is a given number (whose determination is examined in Section 2 below). It is further assumed that:

- Input 1 is a **variable** input.
- Input 2 is a **constrained input**.

1.2.1. Constraint on input 2 and cost minimization

The constraint on input 2 may take a variety of forms.

(i) For example, the firm may be **constrained to use exactly a predetermined amount of input** z_2, denoted $\overline{z}_2$. In this case, the constraint on input 2 can be written as:

$$z_2 = \overline{z}_2$$

and the minimization problem of the firm is thus:

Problem 2 *Choose the input combination* (z_1, z_2) *to produce output* q *which minimizes the production cost* $p_1 z_1 + p_2 z_2$ *subject to the constraint* $z_2 = \overline{z}_2$.

In this problem, the only control variable (i.e. the only adjustment variable) is the quantity of input 1, z_1. It is impossible to use less of input 2 than $\overline{z}_2$. Hence, input 2 is said to be "indivisible". For example, a big furnace used in the production process is an indivisible input because you need to fully use the furnace irrespective of the amount of output.

(ii) When the constrained inputs are divisible, it is always possible for the firm to use less of them. On the contrary, it is impossible, in the short run, to use more of these inputs than what is currently available. Here, the firm faces the following constraint on input 2:

$$z_2 \leq \overline{z}_2,$$

where $\overline{z}_2$ is a **fixed ceiling** on the amount of input 2 currently available. The minimization problem of the firm is thus as follows:

Problem 3 *Choose the input combination* (z_1, z_2) *to produce output* q *which minimizes the production cost* $p_1z_1 + p_2z_2$ *subject to the constraint* $z_2 \leq \overline{z}_2$.

In this minimization problem the production cost is equal to $p_1z_1 + p_2z_2$: if there is a ceiling on the units of input 2 available to the firm, the firm only pays the units of inputs 1 and 2 it actually uses.

(iii) **Fixed cost:** Assume input 2 is divisible as in (ii) above. In practice, the firm:

- might already have contracted to pay $p_2\overline{z}_2$ to use input 2;
- or might already own $\overline{z}_2$ units of input 2. Yet, it would like to use $z_2 \leq \overline{z}_2$ units of input 2. If it is not possible for the firm to sell $\overline{z}_2 - z_2$ in the short run, then it will incur a cost equal to $p_2\overline{z}_2$ (regarding input 2).

In both cases, the production cost is equal to $p_1z_1 + p_2\overline{z}_2$. The firm's minimization problem is thus:

Problem 4 *Choose the input combination* (z_1, z_2) *to produce output* q *which minimizes the production cost* $p_1z_1 + p_2\overline{z}_2$ *subject to the constraint* $z_2 \leq \overline{z}_2$.

Remark 2 *The difference between (ii) and (iii) is that the existence of a fixed input gives rise to a fixed cost in (iii) because the firm must pay* $p_2\overline{z}_2$ *for input* 2 *even if it uses* $z_2 \leq \overline{z}_2$ *in the production process. The fixed cost is equal to* $p_2\overline{z}_2$ *(it is called "fixed" because it is independent of the quantity of input* 2 *used in the production process).*

1.2.2. Solution

We consider the cases (ii) and (iii) defined above.

- **Fixed Ceiling.** In case (ii), the firm's decision problem is to choose the input combination (z_1, z_2) to produce output q which minimizes the production cost $p_1z_1 + p_2z_2$ subject to the constraint $z_2 \leq \overline{z}_2$. There are three subcases:

- When $z_2 < \overline{z}_2$ in the cost-minimizing input combination, there is no change at all compared to the long-run cost-minimization problem. Hence, the short-run cost coincides with the long-run cost that was examined above.
- When $z_2 > \overline{z}_2$ in the long-run cost minimization problem, the short-run and long-run costs can no longer coincide. Indeed, in the short run, the maximum feasible value for z_2 is $\overline{z}_2$. Hence, the constraint on z_2 is binding and reduces the degree of freedom in the production choice. The implication is that in the short-run it is more expensive to produce a given amount of output than in the long term. Therefore, the short-run cost must be higher than the long-run cost.
- When $z_2 = \overline{z}_2$, the short-run and long-run costs are equal.

The link between the short-run and long-run cost functions in case (ii) is shown in Figure 5.

- **Fixed Cost.** In case (iii), the firm's decision problem is to choose the input combination (z_1, z_2) to produce output q which minimizes the production cost $p_1 z_1 + p_2 \overline{z}_2$ subject to the constraint $z_2 \leq \overline{z}_2$. In this case, the firm incurs a "fixed cost" equal to $p_2 \overline{z}_2$ even if it only uses $z_2 < \overline{z}_2$.

  - In particular, the firm must pay $p_2 \overline{z}_2$ even if it chooses an output level equal to zero. Hence, the short-run production cost is equal to $p_2 \overline{z}_2$ for $q = 0$. Given the fixed cost, the short-run production cost is larger than the long-run production cost up to the point where $z_2 = \overline{z}_2$ in the long run.
  - When $z_2 = \overline{z}_2$, the short-run and long-run costs are equal.
  - For $z_2 > \overline{z}_2$, the short-run cost is the same as in case (ii) considered above.
  - When $z_2 = \overline{z}_2$, the short-run and long-run costs are equal.

The link between the short-run and long-run cost functions in case (iii) is shown in Figure 6 (compare with Figure 5).

Notation 1 *For later reference, we call* $S(p_1, p_2, \overline{z}_2, q)$ *the short-run cost of producing y units of output given input prices* p_1 *and* p_2*. (It should be clear from the above analysis that* $S(p_1, p_2, \overline{z}_2, q)$ *is not the same depending on whether we consider case (i), case (ii) or case (iii) introduced above).*

Remark 3 *It is possible to use the procedure explained above for the long-run cost functions in order to derive the graphs of the short-run average cost function and of the short-run marginal cost function.*

Exercise 1 *Derive the short-run cost function when the firm is constrained to use exactly a predetermined amount of input* z_2*, denoted* $\overline{z}_2$ *(called "case (i)" above).*

2. MAXIMIZING PROFITS

In section 1, we did not discuss the objective of the firm. We considered that a rational producer would minimize the cost of producing its chosen output level, irrespective of its objective. We now address the issue of the determination of the output level (which was exogenously given in Section 1). The most standard objective considered in the economic literature is that the producer aims at maximizing its profits and we therefore adopt this objective in this chapter.

2.1. Long-run Profit Maximization

The firm's decision problem is to plan an output and input combination to maximize profits. For convenience, it is assumed that there are only two inputs, labelled 1 and 2. Let p be the price of one unit of output, p_i be the (exogenously) given price of one unit of input i $(i = 1, 2)$ and y be the output level chosen by the firm.

2.1.1. The Firm's Decision Problem

In this setting, the firm's decision problem can be written as follows:

(i) A first way of writing the problem exploits the theory developed in Section 1. It starts from the fact that profits cannot be maximized if production costs are not minimized.

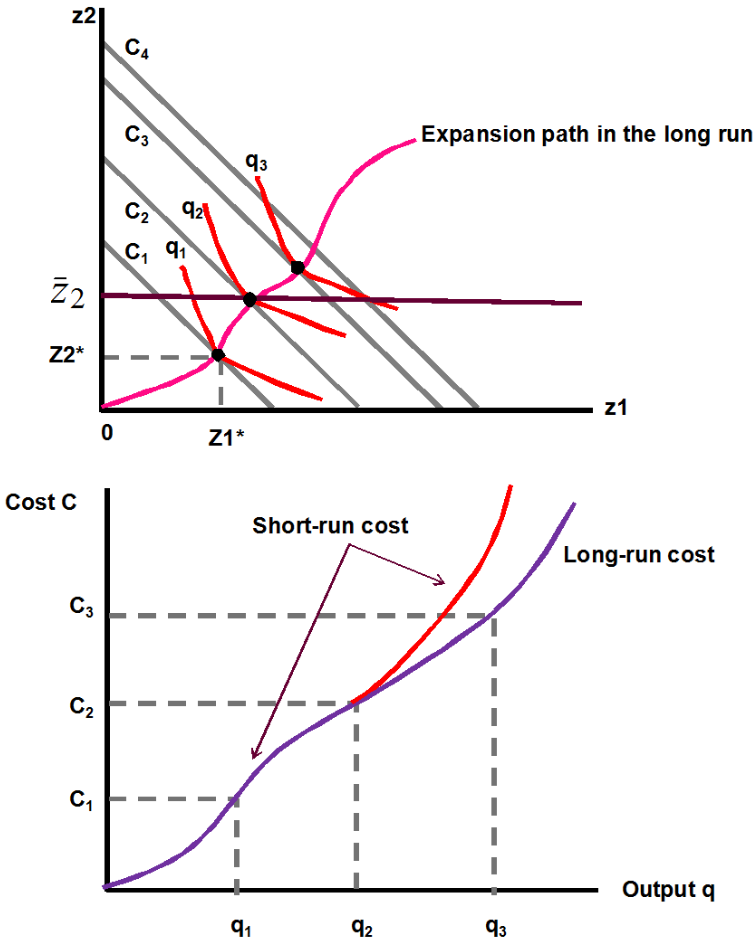

Figure 5: Short-Run and Long-Run Costs Functions (when there is a ceiling on the amount of input 2 that can be used in the short-run but that the firm only pays the units of input 2 it actually uses)

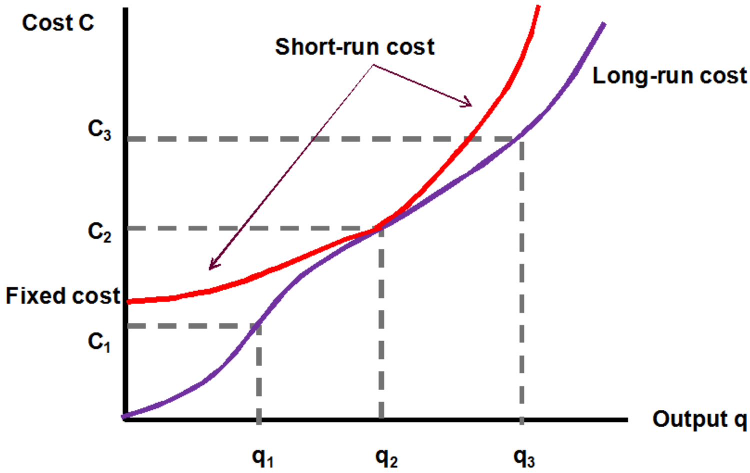

Figure 6: Short-Run and Long-Run Costs Functions (when there is a fixed cost equal to $p_2\overline{z}_2$

Problem 5 (Profits Maximization) *Chose an output level* $y \in \mathbb{R}_+$ *to maximize the firm's profits*

$$\pi := py - C(p_1, p_2, y).$$

The solution to this Problem is an output level y^*. Given y^*, the input levels are obtained from the conditional input demands $z_1^*(p_1, p_2, y^*)$ and $z_2^*(p_1, p_2, y^*)$ defined in Section 1.

(ii) The firm's decision problem can alternatively be stated as follows:

Problem 6 (Profits Maximization) *Chose a production plan* $(y, z_1, z_2) \in \mathbb{R}_+^3$ *to maximize the firm's profits*

$$\pi := py - \sum_{i=1}^{2} p_i z_i$$

subject to the technology constraint

$$y \leq f(z_1, z_2).$$

Note that choosing $y < f(z_1, z_2)$ would be inefficient. Hence, in the optimum production plan, the technology constraint $y \leq f(z_1, z_2)$ must be binding (i.e. it must be written with equality: $y = f(y_1, y_2)$. Problem 6 is therefore equivalent to choosing a production plan $(f(z_1, z_2), z_1, z_2) \in \mathbb{R}^3_+$ to maximize the firm's profits

$$\pi := pf(z_1, z_2) - \sum_{i=1}^{2} p_i z_i.$$

2.1.2. The Firm's Optimum Production Plan

We first use the first formulation (i) of the firm's decision problem. Assuming that $y > 0$, a necessary condition for a maximum is:

$$\frac{\partial \pi}{\partial y} = p - \frac{\partial C(p_1, p_2, y)}{\partial y} = 0 \Leftrightarrow p = \frac{\partial C(p_1, p_2, y)}{\partial y}. \tag{4}$$

In words, this necessary condition means that when the output level is chosen optimally, the cost of producing an extra unit of output is equal to its price. Assume this is not the case. If $p < \partial C(p_1, p_2, y)/\partial y$, then producing an extra unit of output reduces profits by $\partial C(p_1, p_2, y)/\partial y - p > 0$ euros. On the contrary, if $p > \partial C(p_1, p_2, y)/\partial y$, then producing an extra unit of output increases profits by $p - \partial C(p_1, p_2, y)/\partial y > 0$ euros. Hence, the necessary condition (4) that must be satisfied by the optimum output level y^*.

An output level for which $\partial\pi/\partial y = 0$ is called a **"critical point"** in optimization theory. It is critical because it is a candidate for profits maximization. But it could also correspond to the point where profits are minimum. To check that it corresponds to a (local) maximum, we must check that the firm's profit function is locally concave; formally, if we denote by $C'(p_1, p_2, y)$ the marginal cost function, this is the case when

$$\frac{d^2\pi}{dy^2} = -\frac{\partial C^2(p_1, p_2, y)}{\partial y^2} < 0 \Leftrightarrow \frac{\partial C'(p_1, p_2, y)}{\partial y} > 0. \tag{5}$$

This condition is a **"second-order condition for a maximum"**. It tells us that, in an interior maximum[1], the marginal cost curve is strictly increasing. Combined

[1] Consider the optimization problem $\max_{x_1,...,x_n} f(x_1, x_2, ..., x_n)$ subject to the constraints $C_1(x_1) \geq 0$, ..., $C_n(x_n) \geq 0$. The optimum $(x_1^*, ..., x_n^*)$ is interior when all constraints are with strict inequality in the optimum: $C_1(x_1^*) > 0$, ..., $C_n(x*) > 0$. In this case, the optimum is indeed in the interior of the constraint set. In the present example, an optimum is interior if it

with (4), (5) ensures that a critical point is a (local) maximum for the profits function.

Figure 7 illustrates the profit maximization.

- The graph of the profit function $\pi(y)$ is obtained as the difference between the total receipts $R = py$ and the total production cost $C(p_1, p_2, y)$ for all possible output levels y. Here, profits are first negative (up to y_2) and then positive.

- The profit function has two critical points: y_1 and y^* since at both points the marginal profit is equal to zero ($d\pi(y_1)/dy = d\pi(y^*)/dy = 0$).

  – However, y_1 corresponds to a minimum. As shown on the bottom panel, the second-order condition for a maximum (5) is not satisfied at y_1 because the marginal cost curve is then decreasing.

  – In contrast, the second-order condition is verified by y^* which is therefore a (local) maximum (in the Figure y^* is also a global maximum). At the best output level y^*, the marginal cost $C'(p_1, p_2, y^*)$ is equal to the (exogenous) output price. Hence, the marginal receipts p is equal to the marginal cost $C'(p_1, p_2, y^*)$ in the optimum. If this were not the case, it would be possible to increase profits at the margin.

2.1.3. Long-Run Supply Function

Let $p_{\min}$ correspond to the **minimum value of the average cost** (see the bottom panel in Figure 7).

- Now, assume the output price p is just equal to $p_{\min}$. Total receipts py are equal to $p_{\min}y$ and total costs $C(p_1, p_2, y)$ to average cost $p_{\min}$ multiplied by y, i.e. $p_{\min}y$. Hence, profits are then equal to zero.

- If $p < p_{\min}$, then profits are strictly negative and if $p > p_{\min}$ profits are strictly positive.

satisfies $y > 0$.

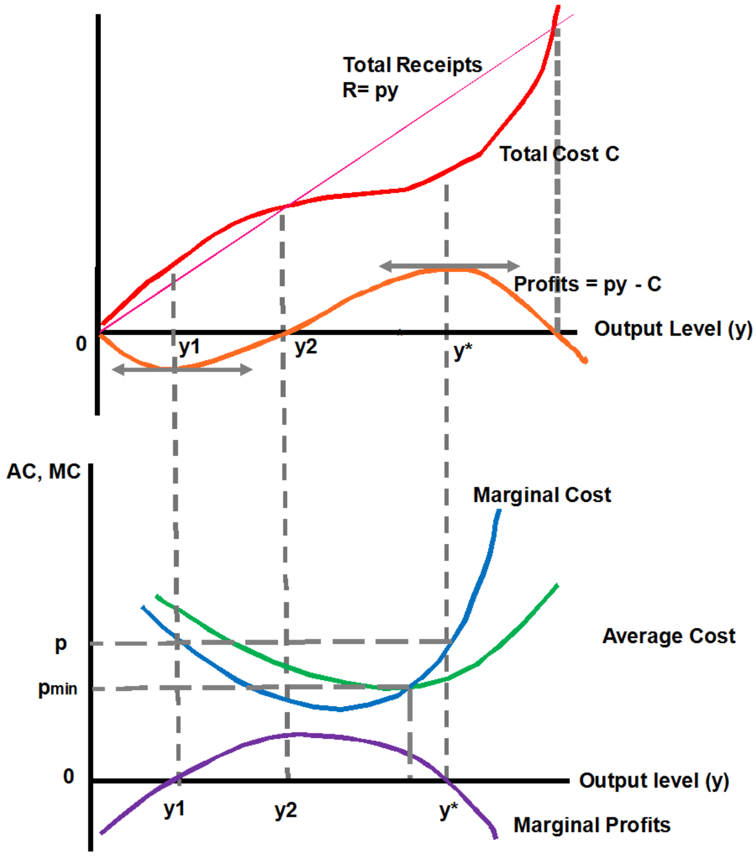

Figure 7: Long-run profit maximization

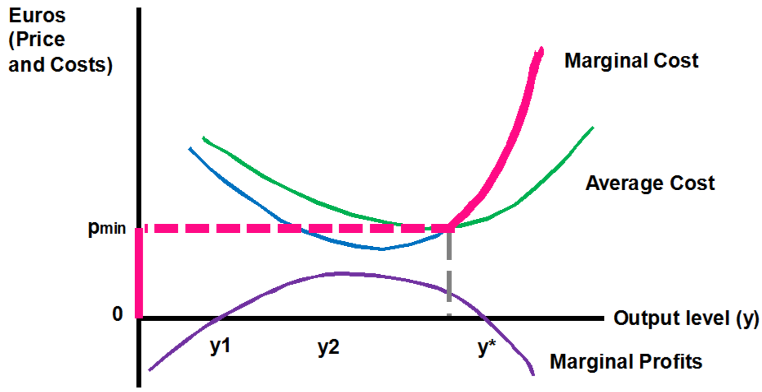

Figure 8: Construction of the Firm's Supply Function (Pink Curve with a discontinuity at $p_{\min}$ shown by the dashed part)

Hence, because $p_{\min}$ is the minimum output price at which the average cost can be covered, a rational producer will not choose to produce (in the long run) if the market price of its output is strictly inferior to its minimum average cost. The implication is that the output level chosen by the firm $y(p_1, p_2, p)$ is equal to zero for all $p < p_{\min}$.

When $p \geq p_{\min}$, the firm chooses a non-negative output level $y(p_1, p_2, p)$. The part of the marginal cost curve above the minimum average cost corresponds to the (output price,optimal output)-combinations.

Therefore, **the (long-run) supply function of the firm - i.e. the optimum output levels given p_1 and p_2 − is first equal to zero for all p below the minimum average cost $p_{\min}$; it is discontinuous at $p_{\min}$ and then coincides with the marginal cost curve.** It is shown in Figure 8.

2.2. Short-Run Profit Maximization

As before, we still consider that z_2 is a constrained input while z_1 is a variable input. The analysis is very similar to that developed in the previous subsection ("Long-run profit maximization"). The difference comes from the fact that the short-run (total) cost function is used instead of the long-run (total) cost function. Recall that $S(p_1, p_2, \overline{z}_2, y)$ is the short-run (total) cost of producing y units of inputs given input prices p_1 and p_2.

The decision problem of the firm is to choose y to maximize profits

$$\pi := py - S(p_1, p_2, \overline{z}_2, y).$$

If $y^* > 0$ is the optimum output level, then the following necessary condition for a maximum must be satisfied:

$$\frac{\partial \pi}{\partial y} = 0 \Leftrightarrow p = \frac{\partial S(p_1, p_2, \overline{z}_2, y)}{\partial y}. \tag{6}$$

In words, this necessary condition means that when the output level is chosen optimally, the cost of producing an extra unit of output *in the short run* is equal to its price. Assume this is not the case. If $p < \partial S(p_1, p_2, \overline{z}_2, y)/\partial y$, then producing an extra unit of output reduces profits by $\partial C(p_1, p_2, y)/\partial y - p > 0$ euros. On the contrary, if $p > \partial S(p_1, p_2, \overline{z}_2, y)/\partial y$, then producing an extra unit of output increases profits by $p - \partial S(p_1, p_2, \overline{z}_2, y)/\partial y > 0$ euros. Hence, the necessary condition (4) that must be satisfied by the short-run optimum output level y^*.

As previously noted, a second-order condition must be verified because, otherwise, profits could be minimum at the candidate y. The second-order condition is simply that the short-run marginal cost must be strictly increasing at the candidate y. In this case, the candidate y is a profit-maximizing output level.

The short-run supply function tells us what is the optimum output level in the short run. Hence, given the input prices and the constraint on z_2, it provides us with the y solution to the short-run profit-maximization problem.

3. CONCLUSION: THE MARKET SUPPLY

This chapter has presented the standard theory of the producer. Two time horizons were considered: short run in which there are constrained inputs and long run in which all inputs are variable. Using the fact that a rational producer always endeavours to minimize the production costs, we have constructed total, marginal and average cost functions in the short run and the long run and emphasized the links between them. In a second step, we have addressed the issue of the determination of the optimum output level. The starting point is to consider the objective of the firm. Here, we have considered that the firm aims at maximizing profits. In this setting, we have constructed the short-run and long-run supply functions of the firm.

The next step in the analysis is to derive the **market supply function**. Let us assume that I firms produce a same output. More precisely, given an output price equal to p (and given the output prices which are the same for all firms), firm i produces $y_i(p)$ units of output $(i = 1, ...I)$. The market supply is obtained by aggregation of the individual supply functions. If we call $Y(p)$ the market supply at price p , we have:

$$Y(p) := \sum_{i=1}^{I} y_i(p) .$$

We have already constructed the demand function of the market, using the theory of the consumer. If there are N consumers with individual demand functions $x_1(p), ..., x_N(p)$, then the market demand is:

$$D(p) = \sum_{i=1}^{N} x_i(p) .$$

For the moment, the output price p is exogenously given. But, given a price p, we know what is the demand and the supply. The next chapter examines **how the output price p is determined**.